DOCUMENT RESUME

ED 368 698 SP 035 125

AUTHOR	Kirsner, Steven A.; And Others
TITLE	Changing Practice: Teaching Mathematics for Understanding. A Professional Development Guide.
INSTITUTION	National Center for Research on Teacher Learning, East Lansing, MI.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	93
NOTE	136p.; A product of the Teaching Mathematics for Understanding Video Project.
AVAILABLE FROM	National Center for Research on Teacher Learning, Michigan State University, East Lansing, MI 48826-1034 (video and guide).
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052) -- Audiovisual/Non-Print Materials (100)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	*Change Strategies; Class Activities; Educational Practices; Elementary School Teachers; Elementary Secondary Education; Faculty Development; *Mathematical Concepts; *Mathematics Instruction; Secondary School Teachers; *Student Role; *Teacher Role; Teaching Guides; *Teaching Methods; Videotape Cassettes
IDENTIFIERS	*NCTM Curriculum and Evaluation Standards; Reform Efforts

ABSTRACT

The materials provided in this package include a 24-minute video and a professional development guide. The video portrays several different approaches to teaching mathematics in ways that are consistent with the National Council of Teachers of Mathematics (NCTM) Standards. The methods employed differ from conventional mathematics teaching in respect to the student's role, the mathematics content, and the teacher's role. The guide is organized into the following sections: (1) Introduction to the Video (the aims and rationale of the video); (2) Using the Video and Professional Development Guide; (3) Orienting Questions for Viewing the Video; (4) Analyzing Classroom Segments on the Video (raises questions and issues in light of NCTM Standards); (5) Annotated Bibliography; (6) Craft Papers (for further study); and (7) List of Publications from the National Center for Research on Teacher Learning (NCRTL). The craft papers in section 6 are: (1) "Changing Minds" (Educational Extension Service/Michigan Partnership); (2) "Could You Say More about That? A Conversation about the Development of a Group's Investigation of Mathematics Teaching" (H. L. Featherstone, and others; and (3) "Assessing Assessment: Investigating a Mathematics Performance Assessment" (Michael Lehman). (LL)

ED 368 698

CHANGING PRACTICE: TEACHING MATHEMATICS FOR UNDERSTANDING

A PROFESSIONAL DEVELOPMENT GUIDE

Developed by

Steven A. Kirsner, John S. Zeuli,
Linda Alford, and Michael J. Michell

The Teaching Mathematics for Understanding Video Project
National Center for Research on Teacher Learning
Michigan State University
East Lansing, Michigan

Summer, 1993

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CREDITS

Changing Practice: Teaching Mathematics for Understanding, a product of the Education Development Center for The National Center for Research on Teacher Learning, Michigan State University, is funded by the Office of Educational Research and Improvement, U.S. Department of Education, and the Educational Extension Service, Michigan Partnership for New Education.

Project Directors: Steven A. Kirsner & John S. Zeuli, Michigan State University
Project Consultant: Linda Alford, Michigan Partnership for New Education
Project Assistant: Michael J. Michell, Michigan State University

Special thanks are due to:

- Deborah Ball, Alyjah Byrd, Sylvia Rundquist, and the students of Spartan Village Elementary School, East Lansing, Michigan
- Anita Johnston and the students of Napoleon High School, Napoleon, Michigan
- Elizabeth Jones, Jane Hannah-Smith, and the students of Dwight Rich Middle School, Lansing, Michigan
- Michael Lehman and the students of Holt High School, Holt Michigan
- Paul Sherbeck and the students of Beaubien Junior High School, Detroit, Michigan
- Lamar Alexander, Secretary of Education
- Shirley Frye, Past President, National Council of Teachers of Mathematics
- Mary M. Kennedy, Director, National Center for Research on Teacher Learning, Michigan State University
- Thomas A. Romberg, Director, National Center for Research in Mathematical Sciences Education, University of Wisconsin-Madison

Special thanks are also due to:

- Daniel Chazan, William Fitzgerald, Glenda Lappan, and Perry Lanier, and Bruce Mitchell, Michigan State University
- Carole B. Lacampagne and Joyce A. Murphy, U.S. Department of Education
- David R. Nelson, Producer/Director, Education Development Center, Newton, Massachusetts

DEDICATION

The ***Changing Practice: Teaching Mathematics for Understanding*** professional development guide is dedicated to Steven A. Kirsner, Project Director, who died before the guide was completed. Steven's vision and energy were responsible for the creation of this guide. Hopefully, uses of it will reflect his spirit—his belief that open-ended discussion and experimentation are the foundation for most forms of human improvement.

John S. Zeuli
Project Co-Director

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SECTION 1: Introduction to the Video

Outlines the aims and rationale of the 24-minute video, Changing Practice: Teaching Mathematics for Understanding.

SECTION 2: Using the Video and Professional Development Guide

Provides suggestions and options for use of the professional development guide.

SECTION 3: Orienting Questions for Viewing the Video

Provides discussion questions about the content of the video. The questions will orient viewers to the provocative ideas in the video and will facilitate closer analysis of classroom segments featured on it.

SECTION 4: Analyzing Classroom Segments on the Video

For classroom segments on the video, raises questions and issues in light of the National Council of Teachers of Mathematics (NCTM) Standards documents. The segments include:

1. Ms. Ball's Third Grade Classroom - Using a Task to Introduce a Unit on Fractions.
2. Ms. Jones's Seventh Grade Classroom - Acquiring a Conceptual Understanding of Decimal Numbers.
3. Mr. Sherbeck's Classroom - Using Writing to Elicit Students' Thinking about Mathematical Ideas.
4. Mr. Lehman's Algebra II Assessment - Assessing Students' Understanding of Algebraic Concepts.

SECTION 5: Annotated Bibliography

Provides a select, annotated bibliography in order to promote further study of the ideas and issues raised in the video and professional development guide.

SECTION 6: Craft Papers

Includes three papers for further study. The first two papers focus on what teachers are thinking, including the struggles they experience, as they attempt to teach mathematics in ways that will enhance students' understanding. A third paper, written by a teacher featured on the video, describes one teacher's attempt to introduce innovative assessment practices in mathematics.

1. Educational Extension Service/MPNE. (1990, Summer). Rethinking mathematics teaching. *Changing Minds*, *1*(1).

2. Featherstone, H., Pfeiffer, L., Smith, S.P., Beasley, K., Corbin, D., Derksen, J., Pasek, L., Shank, C., and Shears, M. (1993). *"Could you say more about that?" A conversation about the development of a group's investigation of mathematics teaching* (Craft Paper 93-2). East Lansing: Michigan State University, National Center for Research on Teaching Learning.

3. Lehman, Michael (1990). *Assessing assessment: Investigating a mathematics performance assessment* (Craft Paper 91-3). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

SECTION 7: List of Publications from the National Center for Research on Teacher Learning (NCRTL)

SECTION 1: INTRODUCTION TO THE VIDEO

> *Teaching methods have to be more for solving problems, reasoning, figuring things out rather than only for memorizing, drilling, and doing things by rote. And that's simply because that's what the world needs today, that's what students have to know.*
>
> Lamar Alexander, Secretary of Education (1992)

Despite this statement and similar ones by members of the educational community, reasoning and problem-solving are not currently the focus of most mathematics instruction. The results of recent national and international surveys of mathematics achievement show that students in the United States have great difficulty understanding mathematical concepts and applying their mathematical knowledge and skills in problem solving situations. Although reformers advocate changes in teaching practice in order to improve mathematics instruction, it is often not made clear what these changes might look like in real classrooms.

What is Happening in the Video?

The video that you are watching portrays several different approaches to teaching mathematics in ways that are consistent with the National Council of Teachers of Mathematics (NCTM) *Standards* documents.

The mathematics teaching portrayed in this video differs from conventional mathematics teaching in three ways: the students' role is different; the mathematics content is different; and the teachers' role is different.

A. Students' Role as Learners of Mathematics

Students are active learners. Young children enter school with a natural curiosity about and some intuitive knowledge of numbers and mathematics. Students should not passively absorb new knowledge acquired in school, but should actively draw on their informal, intuitive knowledge as they internalize new meanings and understandings. The classrooms shown on the video have become, over time, places where students, working individually and in groups, are actively involved in making sense of mathematics, drawing upon, confronting, and expanding their knowledge of mathematics learned in less formal settings.

This portrayal of active mathematics learning is considerably different from what most students have experienced and come to expect in conventional mathematics classrooms. In conventional classrooms, students often work alone and their work consists of practicing routine exercises rather than reasoning or arguing about ideas.

What is proposed in the *Standards* documents is very different. Teachers create a learning environment in which students are engaged in a variety of activities and discussions. Students work through genuine mathematical problems. They reason about mathematics and make connections among important mathematical ideas. A typical mathematics classroom at all grade levels should be one in which students learn to "examine," "transform," "apply," "prove," "communicate," and "solve" mathematical problems.*

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* *Masters have been provided of the curriculum standards in the "Overheads" section of this professional development guide for use as overheads or as handouts.*

B. Mathematics Content

The teachers highlighted in the video think of mathematical content according to the standards in the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989). The vision put forth in the *Curriculum and Evaluation Standards* is that the curriculum serves to build students' mathematical competence and confidence and should represent what mathematicians actually do. Consequently, the curriculum at all levels of schooling focuses on:

(1) Mathematics as Problem-Solving. The curriculum centers on students working through problem situations that help them to investigate and understand mathematical content. Focusing on computational skills and practicing algorithms does not precede students' engagement with problems. Students develop computational skills as they investigate and eventually solve a wide variety of interesting problems appropriate for them.

(2) Mathematics as Reasoning. Students make conjectures, offer evidence, and, in short, build an argument for why they think as they do when working through problems. Students draw on and create models or cite other mathematical facts and relationships to justify their thinking. The focus is on students' reasoning about a problem, not simply getting the right answer.

(3) Mathematics as Communication. Students learn to read, write, and speak the language of mathematics in order to interpret and evaluate mathematical ideas. Students recognize how important it is to use language and symbols to communicate their thinking about mathematical content.

(4) Mathematical Connections. Many opportunities exist in the curriculum for students to see relationships between different mathematical concepts as well as the connections between mathematics and other school subjects.*

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* *Masters have been provided of the curriculum standards in the "Overheads" section of this professional development guide for use as overheads or as handouts.*

C. The Teacher's Role

The video also portrays teachers as taking on a very different role. This new role is more difficult, but ultimately more rewarding for both teachers and their students. Specifically, the teachers in this video have rejected conventional classroom practices that often focus on telling students what to do and then telling them whether they did it right or not. They have slowly moved away from a role that defines them as the only source of authority for right answers and in contrast, have made ongoing progress toward incorporating the fundamental changes advocated in the National Council of Teachers of Mathematics (NCTM) *Standards* documents.

(1) The teachers pose worthwhile mathematical Tasks that determine, and do not simply influence, what students have the opportunity to learn.

(2) The teachers use classroom Discourse as a strategy for helping students make sense for themselves of the mathematics they encounter.

(3) The teachers create and nurture a learning Environment that supports students' reasoning, problem solving, communicating, and connecting ideas in the curriculum.

(4) The teachers engage in systematic Analysis of their teaching and what students are learning as a consequence. Based on the analysis of their teaching, teachers adapt their questions and tasks accordingly.*

Teaching in ways consistent with the *Standards* documents is notably different from what most teachers have been prepared to do and it is not simple to accomplish. Teachers must be committed to rethink their mathematics teaching. They must be willing to rethink what it means to teach mathematics for understanding. Learning to teach for understanding also requires teachers to have colleagues' support and encouragement.

In summary, the mathematics education community has reached consensus about recommendations that call for fundamental change—not simply improvement—in what mathematics students learn and how they learn it. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989), and its companion document, *Professional Standards for Teaching Mathematics* (1991), explain what these important recommendations mean in the areas of mathematics curricula, teaching practices, and teacher preparation. Most importantly, the *Standards*' vision of mathematics teaching and learning is meant for each and every student. The educational community recognizes the power of this vision to enhance the education of all students.

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* *Masters have been provided of the curriculum standards in the "Overheads" section of this professional development guide for use as overheads or as handouts.*

SECTION 2: USING THE VIDEO AND PROFESSIONAL DEVELOPMENT GUIDE

Introduction

The teachers whose classrooms are highlighted in this video are learning to teach mathematics in ways that are consistent with the National Council of Teachers of Mathematics (NCTM) *Standards*. None of the classrooms that appear is meant to represent ideal practice. Instead, we see teachers who are involved in the long-term process of re-orienting their teaching along the lines suggested by the *Standards* documents.

The video and the professional development guide are designed to raise questions and stimulate conversations among educators as well as help teachers to try alternative approaches to conventional practice. Raising questions, experimenting, and engaging in sustained conversations about mathematics teaching and learning are essential if we are to realize the vision of a high-quality mathematics education for all learners.

The video and the accompanying guide will be useful to both teachers who are just starting to rethink their mathematics teaching and those who already possess some knowledge about teaching mathematics for understanding and are now beginning to change the way they teach. The video and guide are appropriate for teachers in many different contexts, such as a mathematics study group, a mathematics department, or at a school in-service. The materials can also be used to introduce other viewers (e.g., prospective teachers, policy makers, and parents) to the *Standards*' vision of mathematics teaching and learning.

In any of these contexts, the showing of the video will be most useful within a long-range plan that revisits, reinforces, and supports teachers changing their mathematics instruction.

When planning to show the tape, it is important to create a setting that is conducive to viewing and discussing the video. Keep in mind that:

- the video is 24 minutes long;
- the room should be comfortable with seating and table arrangements that facilitate the give and take of dialogue; and
- participants will likely need a notebook or handouts to jot down their thoughts as they watch the video and to record their responses to the questions in the guide. Depending on how discussion of the video is orchestrated, discussion leaders also may need chart paper, a blackboard, and a overhead projector and screen.

Uses of the Video and Professional Development Guide

We recommend the following sequence for using the video and professional development guide. These recommendations are intended as helpful suggestions. Discussion leaders may develop imaginative variations that also offer a good foundation for a long-range plan that supports teachers at the local level.

1. General Orientation

Viewers observe and reflect on the video using the "Orienting Questions for Viewing the Video" provided in Section 3. The questions are designed to help a wide range of viewers (teachers, policy makers, parents) focus on important differences between conventional mathematics teaching and the teaching highlighted on the video. Discussion leaders should allocate roughly 1½ to 2 hours to completely work through the orienting questions with the participants. (We have included in the "Overheads" section of this guide masters of all the orienting questions for discussion leaders to reproduce as overheads or as handouts.)

2. Analysis of Classroom Segments on the Video (in light of the *Standards* documents)

Once viewers are oriented to the provocative themes in the video, they will benefit from a closer analysis of the classroom segments featured on it. In Section 4 we have devised discussion questions for four of the classroom segments shown on the video.

We think that each classroom segment will be of interest to teachers no matter what grades they teach. For example, the discussion questions for "Mr. Lehman's Algebra II Assessment" are appropriate for teachers at all grade levels who are interested in innovative assessments of their students' understanding of mathematics. Also, in "Mr. Sherbeck's Classroom," student writing is used to elicit thinking about and encourage discussion of mathematical ideas. Analysis of this segment could lead to broader discussions about using writing to promote meaningful learning of mathematics at any grade level. "Ms. Ball's Third Grade Classroom" could stimulate discussions about using real-life problems in middle-school and high school mathematics classrooms. Consequently, teachers will benefit from working through the discussion questions for each classroom segment.

Discussion leaders may find it useful to show again the classroom segment on the video before or after asking teachers to discuss the questions related it. Also, leaders may want to focus first on the classroom segment that best fits teachers' grade level and then focus on other segments that are also relevant to teaching mathematics for understanding. Discussion leaders may find some overlap between teachers' responses to questions in Section 3 and Section 4. This overlap will help teachers revisit and refocus their attention on significant issues. (Once again, we have included in the "Overheads" sections of the guide masters of the questions for each classroom segment. Discussion leaders can reproduce these masters for handouts or overheads.)

Many discussion leaders have found that one hour (or more) is a useful block of time to allot for discussion of the questions related to each classroom segment. (The number after each of the segments indicates the timed place on the tape where these segments are located. Also, included in the folder on the white pages are the questions for each of the classroom segments.)

	Count Time	
	Begin	End
1. Ms. Ball's Third Grade Classroom	0:48	10:54
2. Ms. Jones's Seventh Grade Classroom	10:55	14:38
3. Mr. Sherbeck's Classroom	14:39	16:39
4. Mr. Lehman's Algebra II Assessment	16:40	20:25

3. Long-Range Planning

After teachers have had extended opportunities to discuss the video in-depth, it is essential that they participate in a long-range plan that revisits, reinforces, and supports them as they change their mathematics instruction.

We have included a select, annotated bibliography in Section 5 of the professional development guide. The annotated bibliography focuses on a wide array of issues related to teaching mathematics for understanding, including teaching and assessing problem solving in mathematics, using writing as a tool to learn mathematics, and helping all students to reason and communicate mathematically. We have also included the National Council of Teachers of Mathematics (NCTM) *Standards* documents in the bibliography (including information on how to order them). We recommend the *Standards* documents as particularly valuable resources for a long-range staff development plan. The NCTM *Curriculum and Evaluation Standards for School Mathematics* provides a detailed vision of what the content, priorities, and emphases of the mathematics curriculum should be for grades K-4, 5-8, and 9-12. The NCTM *Professional Standards for Teaching Mathematics* provides a vision of the kind of teaching needed to support curriculum changes in mathematics. It includes a number of annotated vignettes—drawn from actual classrooms—that will help teachers at all grades levels understand and become more involved in the process of re-orienting their teaching along the lines suggested by the *Standards*.

Discussion leaders may find more immediately helpful the three papers included in Section 6 of the professional development guide. The first two papers include a focus on a teacher featured on the video, Deborah Ball. Both papers further describe several teachers' attempts to grapple with the questions raised by the provocative ideas and practices featured on the video. A third paper, written by a teacher shown on the video, Michael Lehman, describes his attempt to introduce innovative assessment practices in his Algebra II class. It may be helpful for discussion leaders to share these papers with participants for discussion following a viewing of the video and extended discussions of the orienting questions and the questions for each classroom segment.

Also included in Section 7 is a list of publications from the National Center for Research on Teacher Learning (NCRTL). Through its research focus on learning to teach, the NCRTL works to help teachers learn how to create worthwhile learning activities for students, increase students' active involvement in learning, foster students' responsibility for learning, and promote mutual respect and inclusion in the classroom. The Center's publications may also be used to enrich teachers' conversations about the video and may serve as an excellent resource to support a long-range plan for teacher learning.

SECTION 3: ORIENTING QUESTIONS FOR VIEWING THE VIDEO

The following discussion questions are offered in order to stimulate conversation after a viewing of the video. The questions should help the viewer focus on important differences between conventional mathematics teaching and the teaching highlighted on the video. It might be useful for teachers to jot down notes while viewing the video. These notes may include any questions that come to mind during the viewing, as well as responses to the set of questions below. Masters of the orienting questions are included in the "Overheads" section of the guide.

Questions about the Students' Role

(1) When you observe the students in the video,

  (a) What are students saying that you find particularly interesting or considerably different from what students typically say in conventional mathematics classrooms?
  (b) What are students doing that you think is different from what one typically sees during mathematics instruction?

Questions about Mathematical Content

(1) The NCTM *Standards* suggest that students should engage in worthwhile activities. How would you define the content these students are thinking about and how does this content differ from the content offered in your textbooks?

(2) The NCTM *Curriculum and Evaluation Standards* recommends that mathematics curricula at all grade levels focus on (a) mathematics as problem-solving; (b) mathematics as reasoning; (c) mathematics as communication; and (d) mathematical connections. What examples of each curriculum focus are shown on the video?

(3) How do teachers and other educators in the video think about the role and relevance of basic skills and procedures?

Questions about the Teacher's Role

(1) How do the teachers in the video describe what students in their classrooms are saying or doing differently as they learn mathematics?

(2) What kinds of questions do teachers pose and how do they get all students involved in thinking about these questions?

(3) How do teachers guide students' responses and how do they respond to students' unexpected ideas?

(4) Teachers in the video use and describe different approaches to teaching mathematics (e.g., using manipulatives, alternative assessments, student writing, group work).

  (a) Could teachers use these different approaches during mathematics instruction, yet remain tied to traditional mathematics teaching? For example, could a teacher use group work without teaching mathematics for understanding, or use manipulatives without any focus on helping them reason about mathematics?
  (b) If so, what makes these different approaches consistent with teaching mathematics for understanding?

(5) What difficulties do teachers and other educators describe in learning to teach mathematics for understanding?

  (a) What do they describe as the benefits of trying to overcome these difficulties?
  (b) What obstacles might you confront if you want your students to engage in these activities? How might you manage these obstacles in order to help students focus on understanding mathematics?

SECTION 4: ANALYZING CLASSROOM SEGMENTS ON THE VIDEO (in light of the *Standards* documents).*

MS. BALL'S THIRD GRADE CLASSROOM

In 1980, Ms. Ball had been teaching third grade for five years when she began to have doubts about the success of her mathematics teaching. When she moved from third to fifth grade, she found the math more complicated. While she felt that she was still able to explain mathematical ideas clearly, her students did not get the right answers with the same consistency as in previous years. When Ms. Ball talked with the fifth graders about their mistakes, she found that they were not clear about what they were doing and why. She began to talk with her colleagues at the University who steered her toward articles and books about what was wrong with a lot of mathematics teaching and how to do it better. She slowly began to rethink what it meant to learn mathematics and experimented with alternative ways of teaching it. Her interest in mathematics grew. Although she had already started a master's program in reading in Michigan State's College of Education, she changed her focus to mathematics and then pursued advanced graduate studies. She is now an associate professor in the MSU Department of Teacher Education and continues to teach mathematics at Spartan Village Elementary School.

In this third grade classroom, Ms. Ball is about to introduce a unit on fractions. None of her students had formally studied fractions; however, Ms. Ball was certain that they, like most eight-year-old children, had some familiarity and experience with fractions. Therefore, the tasks she selected for this lesson consisted of problems that would serve not only as an introduction for her students, but also as an opportunity for her to assess their informal knowledge (including misconceptions) of fractions.

The video segment begins with Ms. Ball in a classroom posing the following problem about a number of loaves of bread that are to be shared by a given number of people.

Problem: Dena had six loaves of bread. How can she share the loaves equally among twelve people?

As the video continues, students are given time to work on the problem. Students first work individually and then, if they choose, they may work in consultation with the teacher or with other students. Ms. Ball then guides a whole class discussion centering on how students thought about solving the problem.

* *Masters of the questions for each classroom segment are included in the "Overheads" section of the guide.*

Discussion questions:

(1) According to the NCTM *Curriculum and Evaluation Standards*,

> "Students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. The situation should be complex enough to offer challenge, but not so complex as to be insoluble... Learning should be guided by the search to answer questions—first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)." (p. 10)

(a) Do you think that the students in this class were trying to solve a "genuine problem"? What features of the problem make it genuine or not genuine?
(b) How did Ms. Ball use this problem to engage students and elicit their mathematical reasoning and communication?
(c) What could be a genuine problem for students at your grade level?

(2) During the whole class discussion, the teacher rarely gave students any hint about whether she thought they were right or wrong, or even if they were on the right track. Why do you think she behaved this way? What effect did her behavior have on the students' reasoning about math?

(3) One of the purposes for this lesson was to diagnose what students knew about fractions as they began a unit on fractions.

(a) Why might it be important for a teacher to learn what her students know about a topic they had not yet studied?
(b) What did you find out about what students knew? Did anything surprise you?
(c) Contrast this teacher's diagnosis of her students' knowledge of fractions with a more traditional pre-test approach.

(4) Ms. Ball accepted a student's definition of the unit as "cutted bread." Would you do this? Is this good practice? What is your reasoning?

(5) According to the *Professional Teaching Standards*, "The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate." (p. 35)

Orchestrating discourse depends "on teachers' understandings of mathematics and of their students—on judgments about the things that students can figure out on their own or collectively and those for which they will need input." (p. 36)

(a) In what ways did Ms. Ball attend to these suggestions for orchestrating discourse?
(b) Do you see places where a teacher might have made different decisions from those Ms. Ball made? What do you think her decisions were based upon? On what are your decisions based?

(6) The students in this class seemed comfortable reasoning about mathematics in a whole group setting, including at times disagreeing with their classmates. How might a teacher establish these norms of communication in her class?

MS. JONES'S SEVENTH GRADE CLASSROOM

Ms. Jones has taught mathematics to middle school/junior high students for 16 years at Dwight Rich Middle School in Lansing, Michigan. After having taught for 11 years, she had become dissatisfied with the way she taught math. Ms. Jones recalls teaching the operation of fractions. She would provide children with a rule to compute a quotient or product; carefully demonstrate the correct procedure to follow, and expect students to duplicate what she had done. Year after year, the procedure never changed, and just as predictably, children would get to the first or second step in the process, and then forget what to do next. Ms. Jones would reteach the concept and hope students would catch on.

Though Ms. Jones was afraid that she did not know enough about the subject of mathematics, Ms. Jones wanted to learn more. In 1985, she had the opportunity to participate in a local mathematics project. As a participant in the project, she learned more about teaching and learning mathematics than she had learned in all her years of teaching. Though she had to face her own lack of knowledge and misconceptions about some fairly basic mathematics concepts, she learned that relying on learning rules and procedures was not enough. She learned that she needed to ask why, and to continue asking why, until she understood the mathematics that she was expected to teach. In the end, Ms. Jones found that she "was completely transformed into what it meant to be a teacher and learner of mathematics."

In this segment, Ms. Jones is teaching a seventh grade general mathematics class. Having learned that her students do not have a good conceptual grasp of decimals, she is using manipulatives to help her students develop decimal number sense. (Ms. Jones knows that these seventh graders had been exposed to decimals in previous years; she also assumes that their previous experience with decimals likely consisted mainly of paper and pencil computational procedures.) In the class segment that appears on the video, students are asked to use base ten blocks to find different ways of representing a specific decimal number. They work in groups before Ms. Jones guides a whole class discussion.

Discussion questions:

(1) The video segment shows students representing numbers with base ten blocks. According to the *Curriculum and Evaluation Standards*,

> "Students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. The situation should be complex enough to offer challenge, but not so complex as to be insoluble... Learning should be guided by the search to answer questions—first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)." (p. 10)

- (a) Do you think that the students in this class were trying to solve a "genuine problem"? What features of the problem make it genuine or not genuine?
- (b) How could the problem be re-cast to make it more genuine or less genuine?
- (c) How does this method of learning decimals compare with your textbook's treatment of decimals?

(2) The students in this segment worked in groups, as they do regularly in Ms. Jones' class (and as is encouraged by the *Standards*).

- (a) What is the rationale for group work? What benefits do you think students derive from working in groups?
- (b) This class was filmed in the middle of the year. What norms of classroom discourse and behavior do you think Ms. Jones had to cultivate throughout the school year for students to be able to work together as they did in this segment?
- (c) During the class discussion Ms. Jones consistently pushed students to elaborate when they responded to her questions. Why do you think she did this? Compare and contrast how she asked questions with the more traditional questioning that seeks a right or wrong answer from students.
- (d) What other questions or different prompts can you think of to help students develop decimal number sense?

(3) The *Professional Teaching Standards* ask teachers to attend to standards in four areas: selecting worthwhile tasks; creating a learning environment; orchestrating discourse; and engaging in thoughtful analysis. How did this teacher appear to be attending to any of these standards during the lesson?

(4) Four standards in the *Curriculum and Evaluation Standards* are common to all grade levels: mathematics as problem-solving; mathematics as communication; mathematics as reasoning; and mathematical connections. How did these students appear to be attending to any of these standards during the lesson?

MR. SHERBECK'S CLASSROOM

Mr. Sherbeck has taught for 14 years at an urban school, Beaubien Junior High School (grades 7-9) located in Detroit, Michigan. Mr. Sherbeck has always considered himself a teacher who focuses on eliciting and probing students' thinking about subject matter. His undergraduate teacher preparation as a mathematics and science teacher helped him reject conventional classroom practices and oriented him toward a more ambitious view of teaching. Mr. Sherbeck describes the changes in his mathematics teaching over the years as "evolutionary rather than revolutionary." He has continued to learn more about subject matter—not only learning new subject matter but also coming to understand the subject matter he teaches more deeply. He has also had to learn more about innovative teaching practices which help students understand mathematics, such as how to include writing in a math class. When Mr. Sherbeck first started this practice, he asked his students to write about mathematics and then showed their work to his English department colleagues who encouraged and supported his efforts. Mr. Sherbeck quickly saw the value in creating a context where students write about and explain their thinking about mathematics.

In this segment, Mr. Sherbeck has asked students to write down "in a paragraph or two how to construct a 45 degree angle." We see on the video students discussing their reactions to what their classmates have written.

Discussion questions:

(1) The students in this segment worked in groups.

  - (a) What is the rationale for group work? What benefits do you think students derive from working in groups? What are the drawbacks?
  - (b) This class was taped in the middle of the year. What norms of classroom discourse and behavior do you think Mr. Sherbeck had to cultivate throughout the school year for students to be able to work together as they did in this segment?
  - (c) If you wanted your students to talk to each other like the students in Mr. Sherbeck's class, how would you model this kind of conversation? How would you give feedback to students that would encourage constructive comments and discourage off-task comments?

(2) Why might it be important for students to write about their mathematical thinking?

(3) A teacher could ask students to write about mathematics without necessarily contributing to their problem solving, reasoning, or communication skills. How can teachers use students' writing in ways that promote meaningful mathematical learning?

(4) The *Professional Teaching Standards* ask teachers to attend to standards in four areas: selecting worthwhile tasks; creating a learning environment; orchestrating discourse; and engaging in thoughtful analysis. How did this teacher appear to be attending to any of these standards during the lesson?

(5) Four standards in the *Curriculum and Evaluation Standards* are common to all grade levels: mathematics as problem-solving mathematics as communication; mathematics as reasoning; and mathematical connections. How did these students appear to be attending to any of these standards during the lesson?

MR. LEHMAN'S ALGEBRA II ASSESSMENT

Since 1990, Mr. Lehman has worked closely with colleagues—including Michigan State University professors and graduate students as well as his fellow teachers at Holt High School—as part of Holt High School's transition to becoming a "Professional Development School." As a result of this work, Mr. Lehman has made a concerted effort to change his teaching. He realized that his Algebra II students were learning to mechanically manipulate symbols reasonably well, but were not learning to reason, make connections, or communicate about algebraic concepts. He was aware also that traditional paper and pencil tests did not allow him to analyze the kind of teaching and learning for understanding he was trying to promote. Consequently, he incorporated into the semester examination an assessment process that would allow students to demonstrate their understanding to a panel of judges. A segment of this assessment activity appears on the video.

Students had three days of preparation time, in class and on their own time, to work with each other to create solutions to six problems. Students were randomly assigned one of the problems when they met with the panels. The problem to which students on the video responded is on the following page.

Problem: Tom and Patricia Wainwright have decided to start raising Hungarian Longhair White Rabbits to sell as pets. The profit is greater if they sell their rabbits to a wholesaler in quantities of 1000 instead of individual rabbits to individual pet stores. They therefore plan to sell their rabbits in groups of 1000. Tom and Pat plan to start out with 25 rabbits. They know the rabbits will approximately quadruple every month. They will count the next month's populations after they have subtracted the rabbits they sold. The Wainwrights have several questions they need answered before they start this business. Your job is to try to answer them for the Wainwrights. They must be careful not to sell too many rabbits otherwise they would end up having no income for a month or more after they start selling rabbits.

Part 1: The Wainwrights would like to know when they will be able to start making sales and how many groups of 1000 they can sell each month. Caution: Selling and reproduction from one month effect the total population of the next. When figuring the next month's population, do not count the rabbits sold during that month. Could you give them a forecast of their first year of business?

(a) During which month will they be able to start selling rabbits?

(b) Is there a month when they will have to hold back more than 1000 rabbits from market in order to maintain a breeding stock? If so, which month?

(c) During which month will their income even out to a steady income?

(d) If under their current contract, they make $2.50 for each rabbit sold, what will their income be when it finally evens out?

Part 2: The Wainwrights start up cost where $15 for each rabbit they started with, $5,000 for the building to house the rabbits, $250 for the proper permits to run this type of business, and $300 for food for the rabbits.

(a) In which month will they finally break even?

(b) The Wainwrights' monthly expenses are approximately $24,000 per month. How much must they sell their rabbits for in order to continue to average $2.50 per rabbit profit?

Members of the panels which assessed students included teachers, administrators, university faculty, community or school board members, and advanced (i.e. Calculus or Pre-Calculus) high school students. Mr. Lehman ensured that each panel of three contained at least one person who specialized in mathematics. Panelists were encouraged to ask conceptual, sense-making questions instead of procedural ones. Each student had approximately twenty minutes to defend individually his/her solution. Students were encouraged to show panelists written work that supported their solutions. Panelists rated each student according to criteria developed by Mr. Lehman; they had the opportunity to discuss with each other each student's performance and adjust their numerical rating if they were so persuaded. (See the enclosed copy of these criteria and the grading scale.)

Discussion questions:

(1) What are the advantages and disadvantages of this type of assessment, where students individually demonstrate their knowledge and understanding by orally defending their solutions to problems? Contrast what students must know and be able to do for this type of assessment with what they must know and be able to do for more traditional pencil and paper tests.

(2) Although the students prepare their solutions while working in groups, they were required to explain their solutions individually. What might be some advantages, or disadvantages, of allowing groups to demonstrate their solutions collaboratively in this type of assessment activity?

(3) If classroom instruction is not consistent with the type of sense-making, conceptual questions that panelists ask students, then students cannot be expected to do well on the assessment. If you knew your students would be assessed by a panel like this, how would you prepare them? Where would you find problems for them to work on? How would you organize classroom discussions to model this kind of reasoning about mathematics?

(4) What logistical or technical problems might be associated with arranging this type of assessment? What alternatives might be used by teachers who might have difficulty gathering a large number of panelists, but who would like to give students opportunities to demonstrate orally their mathematical sense-making and problem solving abilities?

Algebra II
Performance Assessment

Name ______________________________

Mathematics:

(1)	Making sense of problem (Understanding concepts)	1 2 3 4 5
(2)	Problem solving strategies (Methods used)	1 2 3 4 5
(3)	Accuracy of results	1 2 3 4 5
(4)	Interpreting results (What do the results mean?)	1 2 3 4 5

Presentation:

(1)	Ability to communicate results (Clarity, use of charts/graphs)	1 2 3 4 5
(2)	Explanation (Able to answer questions)	1 2 3 4 5

Grading Scale

26 - 30	A
22 - 25	B
18 - 21	C
15 - 17	D

Overall Score

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Comments:

SECTION 5: ANNOTATED BIBLIOGRAPHY

CHANGING PRACTICE: TEACHING MATHEMATICS FOR UNDERSTANDING

Ball, D. L. (1990). *Halves, pieces, and twoths: Constructing representational contexts in teaching fractions* (Craft Paper, 90-2). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Learning to teach mathematics for understanding is not easy. First, practice itself is complex. Second, many teachers' traditional experiences with and orientations to mathematics and its pedagogy are additional hindrances. This paper examines the territory of practice and reviews some of what we know about those who would traverse it—prospective and experienced elementary teachers. In analyzing practice, the author focuses on one major aspect of teacher thinking in helping students learn about fractions: the construction of instructional representations. Considerations entailed are analyzed and the enactment of representations in the classroom is explored. The term representational context is used to call attention to the interactions and discourse constructed in a classroom around a particular representation. The author provides a window on her own teaching practice in order to highlight the complexity inherent in the joint construction—with students—of fruitful representational contexts. The paper continues with a discussion of prospective and experienced teachers' knowledge, dispositions, and patterns of thinking relative to representing mathematics for teaching. The author argues that attempts to help teachers develop their practice in the direction of teaching mathematics for understanding requires a deep respect for the complexity of such teaching and depends on taking teachers seriously as learners.

Ball, D. L. (1991, September). Beginning a conversation about the NCTM *Professional Standards for Teaching Mathematics*: Improving, not standardizing, teaching. *The Arithmetic Teacher*, *39*(1), 18-22.

The first in a series of "professional conversations" about the National Council of Teachers of Mathematics (NCTM) *Professional Standards for Teaching Mathematics*, this article begins with a general discussion of the standards, explaining that they do not reduce teaching to recipes; rather, they express a set of shared professional values and goals for mathematics teachers and help guide teachers' efforts to teach mathematics well. The author provides an overview of the document's first section, "Standards for Teaching Mathematics," including a discussion of the terms *tasks*, *discourse*, *environment*, and *analysis*. Then, focusing specifically on the standard for selecting and using good learning tasks for students, she analyzes the strengths and weaknesses of three sample problems to help teachers determine what a worthwhile mathematical task entails.

Ball, D. L. (1991, November). What's all this talk about "discourse"? *Arithmetic Teacher*, *39*(3), 44-48.

The author defines "discourse" as intended by the standards. Using a discussion drawn from her own third-grade-class and entries from her teaching journal, the author illustrates how thoughtful discourse can help students learn to discuss mathematics.

Baroody, A. J. (1987). *Children's mathematical thinking*. New York: Teachers College Press.

In this book, the author explains how young students (K-3) think and learn about mathematics. He argues that what we know about how children think mathematically is inconsistent with what he terms the "absorption theory of learning." Instead of passively absorbing new knowledge, children actively build on their intuitive, informal knowledge as they produce new meanings and understandings. The author also suggests teaching practices which are more consistent with how students learn mathematics.

Charles, R. I., & Lester, F. (1982). *Teaching problem solving: What why and how*. Palo Alto, CA: Dale Seymour Publications.

This book addresses what mathematical problem solving involves, why problem solving is so important, and how teachers can make problem solving an integral part of mathematics instruction. After defining and discussing what is and is not a problem, the authors present many useful guidelines and suggestions for teachers who want to make problem solving the focus of their mathematics programs.

Charles, R. I., & Silver, E. A. (1988). *The teaching and assessing of mathematical problem solving* (pp. 371-375). Reston, VA: National Council of Teachers of Mathematics.

This research-oriented collection of essays is directed to all who play a role in the reform of mathematics teaching (math educators, teachers, administrators, etc.). Selections by Resnick, Schoenfeld, Lester, Silver, Carpenter, Noddings, and others represent perspectives from several fields. All are concerned with the teaching and evaluation of problem solving. Student collaboration, alternative assessment, motivating students, student-centered instruction, the use of a constructivist approach to teaching and learning, and problem solving for all students are just some of the issues highlighted.

Chazan, D. (1992, May). Knowing school mathematics: A personal reflection on the NCTM's teaching standards. *Mathematics Teacher*, *85*(5), 371-375.

Chazan talks about his struggle to think about math and math teaching in new ways. The question of *why* the quadratic formula works to identify the solutions of quadratic equations was student initiated. The question inspired Chazan to pursue understanding the "whys" and not just the "how to's" in learning and teaching mathematics. Chazan shares the process of his realization that if educators are to teach in a meaningful and non-manipulative way, helping students to recognize the uses of math in their daily pursuits, then teachers need to know math in that way as well. "A way I was not taught in," says Chazan.

Cocking, R. R., & Mestre, J. P. (Eds.). (1988). *Linguistic and cultural influences on learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

A major premise of the *Standards* is that all children can learn, regardless of cultural background. This book explores issues critical to educating *all* children. Some of the topics addressed are languages of minority children, bilingualism, and studies of specific cultural groups such as Asian-Americans, Mexican-Americans, and Native Americans.

Connolly, P., & Vilardi, T. (Eds.). (1989). *Writing to learn mathematics and science*. New York: Teachers College Press.

The *Standards* emphasize writing as a tool for learning mathematics. One of the video segments highlights one use of writing in a middle school setting. This book contains several chapters that illustrate a spectrum of options and rationales for using writing at all levels of mathematics teaching.

Cooney, T. J., & Hirsch, C. R. (Eds.). (1990). *Teaching and learning mathematics in the 1990s: 1990 Yearbook*. Reston, VA: National Council of Teachers of Mathematics.

This highly accessible volume is rich with practical ideas for teachers of mathematics at every level. The 28 essays are organized in seven sections: the relationship between research and practice, effective methods for teaching mathematics, assessing students' mathematical understanding, cultural factors in teaching and learning, contextual factors in teaching and learning, technology, and professionalism and its implications for teaching and learning. Each essay summarizes relevant research and includes practical suggestions for teachers at both elementary and secondary levels.

Cramer, K., & Bezuk, N. (1991, November). Multiplication of fractions: Teaching for understanding. *Arithmetic Teacher*, *39*(3), 34-37.

The authors use the "Lesh translation model with five modes of representation" to illustrate how a multiplicity of approaches help students to better understand mathematical concepts. Many activities for teaching multiplication of fractions are used as models for different modes of representation.

Grouws, D. A., Cooney, T. J., & Jones, D. (Eds.). (1988). *Effective mathematics teaching* (Vol. 1). Reston, VA: National Council of Teachers of Mathematics.

This collection of papers reports on what research has to say about what effective mathematics teaching is and how to foster it. The papers discuss a variety of issues that must be addressed in building a framework for effective mathematics teaching.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. *Harvard Educational Review*, *55*(2), 178-194.

The author is a university mathematics educator and an elementary school teacher. As a practitioner striving to improve her craft and the learning environment of her elementary school classroom, she views the teacher as a dilemma manager turning conflict into teachable moments. In this article she analyzes two pedagogical dilemmas. The first case is drawn from her own fifth grade class. Lampert addresses her discovery of how the misbehavior of her male students created a situation of gender inequity in her classroom. The second case focuses on a conflict over the nature of knowledge arising in a fourth grade science lesson.

Maher, C. A., & Martino, A. M. (1992, March). Teachers building on students' thinking. *Arithmetic Teacher*, *39*(7), pp. 32-37.

This article touches on several important issues related to teaching math for conceptual understanding. The authors use dialogue between two girls working "independently yet cooperatively" on a real world problem. The classroom teacher enters into the conversation to extend the girls' problem solving discussion by adding a new dimension to the problem. The authors illustrate that holding back the answer can be an effective instructional strategy.

Mathematical Sciences Education Board. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

This report issues a comprehensive call for reform in mathematics education, from kindergarten through graduate school. It explains the seriousness of the problem, charts a general course for the future, and outlines a national strategy for pursuing that course. Discussing mathematics as the key to economic opportunity, the link between equity and excellence in education, the nature of mathematics, new standards for curriculum and assessment, new approaches to teaching, and the impact of calculators and computers, *Everybody Counts* provides an overview of the current situation, necessary reforms, and obstacles to those reforms. Arguing that high expectations yield greater achievement, the authors call for all students to learn a significant core curriculum. They also propose changing from a "transmission of knowledge" model of teaching mathematics to "student-centered practice" and from inculcation of routine skills to "developing broad-based mathematical power."

Mathematical Sciences Education Board. (1990). *Reshaping school mathematics. A philosophy and framework for curriculum*. Washington, DC: National Academy Press.

This booklet proposes a framework for reform of school mathematics curricula in the United States. It is intended to complement *Everybody Counts* and the NCTM *Curriculum and Evaluation Standards*, and it emphasizes two fundamental issues discussed in those documents: (1) changing perspectives on the need for mathematics, the nature of mathematics, and the learning of mathematics; and (2) changing roles of calculators and computers in the practice of mathematics. Philosophical issues are treated in much greater depth than in either of the other two documents and extensive reference is made to current research on topics such as technology in education and teaching for higher-order thinking. The booklet also proposes a set of fundamental principles, specific goals, and enabling conditions for the new mathematics curriculum.

McDiarmid, G. W. (1992). *Kathy: A case of innovative mathematics teaching in a multicultural classroom*. Fairbanks: University of Alaska Fairbanks, Center for Cross-Cultural Studies.

This case study presents a clear portrait of a math teacher who bases her instruction on achieving mathematical understanding rather than learning mathematical rules. Her methods are challenged by an African-American parent who wants her daughter to learn mathematics and do well on standardized tests. Kathy's story illustrates how all students, especially children of color, benefit from the innovative instruction she uses in her classes.

McIntosh, M. E. (1991, September). No time for writing in your class? *Mathematics Teacher*, *82*(6), 423-433

The author distinguishes between math teachers teaching writing, in the spirit of writing across the curriculum, and using "writing to learn." The article provides clear descriptions of four written forms (logs, journals, expository, creative) that can help students learn, help the teacher assess student learning, and put math in more "human terms."

National Council of Teachers of Mathematics. (1989, March). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Reflecting a broad consensus within the mathematics education community, this document establishes a comprehensive framework to guide reform in school mathematics in the 1990s, providing a detailed vision of what the content, priorities, and emphases of the mathematics curriculum should be. The main body of the document consists of curricular standards for grades K-4, 5-8, and 9-12, and evaluation standards. The curricular standards articulate the following goals for all students: (1) that they learn to value mathematics; (2) that they become confident in their ability to do mathematics; (3) that they become mathematical problem solvers; (4) that they learn to communicate mathematically; and (5) that they learn to reason mathematically. Some of the standards address these goals directly; others address a specific topic—such as whole number computation or statistics—in the context of the goals. All articulate a vision of the classroom as a place where students learn important mathematical ideas by actively engaging in conjecturing, reasoning, and exploring interesting problems; using manipulatives, computers, calculators, and other hands-on tools; and working often in cooperative groups. The evaluation standards address assessment strategies, judgements about students' progress, and evidence about the quality of mathematics programs. They stress using assessment to guide instruction, aligning assessment with the curriculum, and taking advantage of multiple methods of assessment.

To order, call (800) 235-7566 or (703) 620-9840.

National Council of Teachers of Mathematics. (1991, March). *Professional standards for teaching mathematics*. Reston, VA: Author.

As part of the NCTM's broad framework for reforming school mathematics, these standards provide a vision of the kind of teaching necessary to support the curriculum proposed in the *Curriculum and Evaluation Standards*. This document rests on the assumptions that teachers are key figures in changing mathematics education and that teachers need long-term support and adequate resources to effect the proposed changes. It includes sections on teaching mathematics, evaluating teachers, professional development, and institutional support for teachers. Annotated vignettes, drawn from actual classrooms, are used throughout to illustrate teachers' interactions with colleagues, students, supervisors, and university faculty. Running through the entire document are characterizations of five major shifts to be made in the environment of mathematics classrooms: (1) toward classrooms as mathematical communities (away from classrooms as collections of individuals); (2) toward logic and mathematical evidence as verification (away from the teacher as sole source of right answers); (3) toward mathematical reasoning (away from just memorizing procedures); (4) toward conjecturing, inventing, and problem solving (away from mechanical manipulation and symbols); and (5) toward connecting mathematics, its ideas, and its applications (away from viewing mathematics as isolated concepts and procedures).

To order, call (800) 235-7566 or (703) 620-9840.

Parker, R. E. (1991, September). Implementing the *Curriculum and Evaluation Standards*: What will implementation take? *Mathematics Teacher*, *82*(6), 442-448, 478.

The author describes the kind of instruction implied by the standards in the context of two problems, emphasizing the importance of "good content" to creating rich problem solving experiences. Student work on these problems is included to illustrate the approach taken and results achieved. The article also includes a brief discussion of the kind of support teachers will need in order to change their practice.

Peterson, P. L., & Knapp, N. F. (1992). Inventing and reinventing ideas: Constructivist teaching and learning. In G. Cawelti (Ed.) (in press). *The 1993 yearbook of the Association for Supervision and Curriculum Development (ASCD)*. Washington, DC: ASCD.

The authors begin this chapter by "unpacking" the constructivist view of knowledge and learning. Two case studies of constructivist mathematics teaching and learning in elementary classrooms are presented. Through the case study approach, the perspectives and voices of the two teachers have been preserved. Four critical questions are pursued here: What is the teacher's role in constructivist learning? How can the teacher honor both student-constructed knowledge and traditionally-accepted knowledge? How can teachers involve diverse students in community problem-solving? How can teachers help students handle the risks of publicly sharing and debating ideas?

Resnick, L. (1987). *Education and learning to think*. Washington, DC: National Academy Press.

This monograph addresses the question of what schools can do to promote the teaching and learning of what are called "higher-order skills." The author presents a working definition of higher-order thinking skills and discusses these skills in the context of various subjects, including mathematics. Resnick challenges the common assumption that there is a natural learning sequence that moves from lower level activities ("basic skills") to higher level ones. This assumption, which is used to justify years of drill on the *basics* before thinking and problem solving are introduced, is contradicted by the cognitive research that Resnick discusses. She says that "the most important single message of modern research on the nature of thinking is that the kinds of activities traditionally associated with thinking are not limited to advanced levels of development. Instead, these activities are an intimate part of even elementary levels of reading, mathematics, and other branches of learning." Moreover, failure to cultivate aspects of thinking, she argues, may be the source of major learning difficulties, even in elementary school. Resnick also offers suggestions for teaching and organizing instruction in higher order thinking.

Resnick, L. & Klopfer, L. E. (Eds.). (1989). Toward the thinking curriculum: Current cognitive research. *1989 Yearbook of the Association for Supervision and Curriculum Development*. Alexandria, VA: ASCD.

This collection provides an overview of recent research by cognitive psychologists on the mental processes underlying thinking and learning in the classroom. Seven of the nine chapters focus on the thinking processes that contribute to learning in mathematics, science, reading, and writing. These chapters include recommendations for curriculum and instruction in these subject areas. Of particular note are Chapter 1, an overview of cognitive research findings, and Chapter 9, a discussion of the implications of research for instructional practices.

Romberg, T. A. (1992). Assessing mathematics competence and achievement. In Berlak, H., Newmann, F. M., Adams, E., Archibald, D. A., Burgess, T., Raven, J., & Romberg, A. (Eds.), *Toward a new science of educational testing and assessment* (pp. 23-52). Albany, NY: SUNY Press.

In this article, Romberg reviews the rationale for reforming the teaching and learning of mathematics, and analyzes current testing procedures. He concludes that current testing is based on an outdated set of assumptions about mathematical knowledge, learning, and teaching, and that its continued use will be counterproductive to needed reform. He presents principles upon which new assessment instruments should be based and gives examples of tasks that are consistent with these principles.

Romberg, T. A., & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. C. Wittrock (Ed.), *Handbook of research on teaching: A project of the American Educational Research Association* (3rd ed., pp. 850-873). New York: Macmillan.

In this chapter, the authors review the recent shift in direction of research on students' mathematical learning and thinking and discuss what this shift implies for research on mathematics teaching. They begin by examining the current state of instruction in mathematics, and they challenge the assumptions about student learning on which current instruction is based. After analyzing more recent research on mathematics teaching and learning, the authors discuss the new directions that they think research in mathematics education should take.

Schoenfeld, A.H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist, 23*, 145-166.

The author conducted observations over an entire school year of a "good" geometry class, taught by a well-regarded teacher whose students received good grades and continued on to college. He shows that much of what students learned in this traditional setting is inconsistent with the type of learning that is advocated by the standards. This article raises important and provocative questions about what kind of mathematical knowledge students really learn, even in traditionally "well taught" mathematics classes.

Silver, E. A., Kilpatrick, J., & Schlesinger, B. (1990). *Thinking through mathematics: Fostering inquiry and communication in mathematics classrooms*. New York: College Entrance Examination Board.

This booklet explains changing perspectives on what it means to learn and do mathematics and explores how these perspectives can be incorporated into the teaching of secondary school mathematics. Using numerous examples of classroom situations, the booklet aims to stimulate teachers to connect thinking and mathematics, moving toward classrooms in which students construct their own mathematical understanding, justify conjectures, solve open-ended problems, and create their own problems. The authors suggest ways in which teachers can encourage greater communication and inquiry in their classes by modifying textbook exercises, incorporating non-routine problems, using group work and written logs, and making other minor but thought-provoking changes in the ordinary curriculum. The booklet contains much practical advice for teachers beginning the difficult, but rewarding, process of changing the mathematics classroom, including advice on goal setting, identifying where to make changes, and collaborating with other teachers.

Steen, L. A. (1990). *On the shoulders of giants: New approaches to numeracy*. Washington, DC: National Academy Press.

Written for educators, curriculum developers, educational policymakers, and parents, this book is intended to stimulate creative approaches to mathematics curricula and prompt a redefinition of what is basic or fundamental in mathematics education for the twenty-first century. The volume includes five essays—each written by a different mathematician —presenting mathematics as the language of patterns and offering imaginative ways of developing mathematical ideas from kindergarten through college. The topics—dimension, quantity, uncertainty, shape, and change—provide examples of rich mathematical ideas around which the mathematics curriculum might be reorganized. Themes such as measurement, symmetry, visualization, and algorithms connect the essays. The topics covered, which are at the forefront of current mathematics research, may be unfamiliar to many teachers, but are adaptable to classroom teaching. The authors bridge the gap between school and real mathematics.

Stenmark, J.K. (Ed.). (1991). *Mathematics assessment: Myths, models, good questions, and practical suggestions.* Reston, VA: National Council of Teachers of Mathematics.

This booklet, written for teachers, provides a collection of assessment techniques that focus on student thinking. The booklet stresses the importance of linking assessment closely to instruction and the need for authentic assessment tasks that bridge the gap between school and real mathematics. It describes the advantages for both students and teachers of assessment alternatives; identifies and counters a number of myths about assessment (for example, that there is always a single answer to a math problem); reviews current assessment practices and identifies needed changes; and offers a sample plan for developing a new approach to assessment. At the heart of the booklet are three sections on alternative assessment approaches: performance assessment; observations, interviews, conferences, and questions; and mathematics portfolios. Each of these sections contains guidelines for developing and using the assessment techniques including classroom management and logistics; sample tasks, questions, and forms; and evaluation criteria. The booklet concludes with a section on implementing new models of assessment, that focuses on documentation and demonstration of validity, along with guidelines for student self-assessment.

Stigler, W. S., & Stevenson, H. W. (1991, Spring). How Asian teachers polish each lesson to perfection. *American Educator*, *15*(1), 12-20, 43-47.

This article, which is based on a forthcoming book entitled *Cultural Lessons: A New Look at the Education of American Children*, illustrates how mathematics teaching in Asian elementary classrooms is more engaging than in American classrooms. Asian teachers seem to "polish each lesson to perfection." Lessons are coherent and the teacher acts more as a "knowledgeable guide, rather than . . . dispenser of information and arbiter of what is correct." Of the many differences between what the authors observed in Asian and American classrooms, the discussion of mathematical concepts in Asian classrooms is of particular interest; questions posed in Asian classrooms are designed to stimulate thought, not simply to get a correct answer. Most relevant to American concerns of educational equity is how Asian teachers deal with diversity of students backgrounds. Asian elementary children are not tracked or grouped by ability. In a whole-group instructional setting, Asian teachers vary their approaches to teaching and presenting materials within a lesson to better engage students who come to the setting with diverse past experiences.

Willoughby, S. S. (1990). *Mathematics education for a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Willoughby, a mathematician and educator, discusses ways teachers and supervisors can change the mathematics education of children, focusing on the most striking differences between current and suggested practice. He maintains that mathematics instruction should move from the concrete to the abstract, neglecting neither; that unreal problems do more harm than good; and that children should be taught to use calculators properly. Throughout the book, Willoughby provides examples of specific activities and curriculum suggestions. For instance, to illustrate making "vertical connections" throughout the curriculum, as recommended in the NCTM *Curriculum and Evaluation Standards*, he suggests activities for teaching functions at every grade level. These range from kindergartners creating a "computer" out of a cardboard box, to third graders using circle-and-arrow diagrams, to sixth graders using graphing computers and calculators. The book concludes with a discussion of ways to support teachers in bringing about the kinds of changes advocated.

Summer 1990
(Special Double Issue)

changing minds

A publication of the
Michigan Educational
Extension Service

Rethinking Mathematics Teaching

When teachers in the Professional Development Schools began to talk to one another about what they wanted to rethink and revise in their schools and classrooms, the subject of mathematics teaching came up over and over again. Some winced as they remembered the math classes of their childhood and worried that they were doing little better than their own teachers had done. They were not sure that their own students either enjoyed or fully understood the mathematics that they did in school. The depth and breadth of this concern led teachers in several schools to organize math study groups.

This first issue of *Changing Minds* describes some particular ways of teaching math, some of the thinking that PDS teachers, undergraduates, and graduate students in education are doing with regard to math teaching, and some of the things that teachers are trying as they attempt to teach mathematics in ways that will enhance their students' understanding. We hope that these glimpses of students and teachers grappling with the questions raised by this sort of teaching will help others see what is involved in changing the teaching of math, and will tempt some teachers to look again at their own teaching, and to join these adventurers as they explore and experiment.

The issue begins with a look at two new publications of the National Council for the Teaching of Mathematics which call for sweeping changes in mathematics teaching. It then focuses on Deborah Ball, a teacher for 15 years at Spartan Village Elementary School in East Lansing, an assistant professor at the Michigan State University College of Education, and an important voice in the national conversation about the teaching of mathematics.

After an introduction to Ball's classroom and to her students, her practice, and her thinking, we listen in on two groups as they reflect on what they have seen of Ball's teaching. The first group is a class of college students who are considering careers in teaching. These young men and women have read a chapter of *Active Mathematics Teaching* by Thomas Good, Douglas Grouws, and Howard Ebmeier and are comparing the

Changing Minds...

This is the inaugural issue of *Changing Minds*, a publication of the Michigan Educational Extension Service (EES). The title is intended to convey the several senses in which contemporary education in Michigan is a matter of changing minds.

First, we are changing our minds about the kind and level of education required for today's students to prosper in a dramatically changed economy, in a scientifically and technologically advanced democracy, and in a society that is growing more diverse along just about every imaginable dimension – ethnically, culturally, linguistically, in family patterns, in the integration of handicapped people into mainstream institutions, and in numerous other ways.

Continued on page 2.

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Changing Minds continued from page 1.

To build a robust twenty-first century economy, we need people who can go beyond the rote application of standard procedures and bureaucratic rules. We need people who can think, who can solve novel problems on the job, who can work in teams with diverse membership, who can communicate clearly both orally and in writing, who understand the mathematical and scientific ideas underlying complex systems, who take responsibility for themselves and for achieving results, and who want and know how to go on learning throughout their lives. Many of these same forms of knowledge and know-how are also required for active citizenship at all levels, as well as for life as a consumer, a family member, and in other phases of our lives.

Second, we now know that learning itself is a matter of changing your mind. Real learning, learning of the sort required by the emerging economy and society, consists not in the passive accumulation of facts and skills, but in active construction and reconstruction of ideas, of mental models of how things work, of what written material really means, and of the mental tools we refer to as skills, such as the skills involved in reading, writing, and mathematics. Learning a scientific idea – the notion that when we push on an object the object pushes back, for example – frequently means revising our commonsense or intuitive understandings of the world around us, not just memorizing a Newtonian law. The same is true of learning about history, about social systems, about literature, about mathematics.

Third, for most experienced as well as new teachers, learning to teach in a way that fosters such constructivist learning involves changing their minds about how learning really takes place and how they can stimulate, support, and orchestrate it. For a teacher who understands that learning is construction rather than absorption, the classroom is a room full of minds. A room full of people who bring different strengths, preconceptions, learning strategies, languages, values, and perspectives to the material to be learned and therefore see it in different ways. A room full of people who may learn in different ways, but all of whom can learn. At some level, all teachers know this, but too few of us are aware of the conflict between what we say we believe and how we actually behave. We have to re-examine our beliefs, and our practice in light of our beliefs. Changing our practice involves changing our minds.

Fourth, we have to change our minds about schools and universities as institutions and about the relationships between them. If students are to be active thinkers and learners, then we must rethink teachers' work, as well. We cannot view teachers as industrial age production workers, carrying out routine tasks under close supervision

Continued on page 23.

traditional style of teaching algorithms recommended by the authors with what they see Ball doing. They feel comfortable with the predictable, teacher-controlled teaching Good and his colleagues describe; they react with a mix of feelings to Ball's very different approach, with its emphasis on the reasoning of eight-year-olds. As one of them says, "We felt she could give them more direction, lead them to something, because they were just spending a lot of time discussing zero." The second group is the Math Study Group in Elliott Elementary School in Holt: five experienced teachers, two "interns" who have recently graduated from Michigan State and are beginning their professional careers by teaching math in this professional development school, one graduate student, and two professors from the College of Education.

We then move back to the classroom and watch Ruth Heaton, a veteran teacher and a graduate student, as she begins to change the way she teaches mathematics. Ruth is working with Magdalene Lampert, a colleague of Deborah Ball both at the College and at Spartan Village Elementary, and also a nationally recognized expert on mathematics education. As they discuss and examine Ruth's changing practice, Maggie and Ruth are learning about challenges that traditional teachers face as they try to teach mathematics in this unfamiliar way.

The last two articles, "Special Circumstances: Rethinking Middle School Math" and "Math Projects at Holt High School," take us into math classes in two secondary schools: Holmes Middle School in Flint and Holt High School.

All the teachers described in these pages have studied mathematics in traditional classrooms. All have taught in traditional ways, explaining the rules for conventional mathematical procedures and helping their students to master these algorithms through repeated practice on a variety of problems. As thoughtful students and teachers encountering an approach to teaching which differs dramatically from the images of mathematics teaching they carry in their heads, they are looking at their own teaching in a different way, and grappling with what it might mean to try to change their practice. ■

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A Call to Change the Teaching of Mathematics

A year ago the National Council of Teachers of Mathematics released a report calling for sweeping changes in mathematics curricula in American schools and in the ways in which we assess our children's knowledge of mathematics.[1]

The Council asks us to imagine ways of teaching and learning math which are quite different from those that most of us encountered in our own schooling, and very different from the picture that develops in our minds when we hear a teacher telling her class that the time has come for math. Most teachers, both now and in the past, have first taught children algorithms – e.g. "to divide by a fraction, invert and multiply" – and then assigned them numerical calculations on which to develop skill and speed in using this rule before moving on to the (often hated) "story" problems at the end of the chapter.

The Council proposes reversing this strategy, arguing that "knowledge should emerge from experience with problems." In other words, if children encounter an interesting problem *before* they have learned the arithmetic operation which adults would use to solve such a problem, they will invent for themselves – if it is the right problem at the right time – a variety of ways to to solve the problem. "'Knowing' mathematics is 'doing' mathematics," says the Council. "A person gathers, discovers, or creates knowlege in the course of some activity having a purpose."

But although the Council, in their 1989 volume on curriculum and evaluation standards, envisions elementary schoolers expanding their mathematical knowledge through investigations of interesting real world problems, neither popular elementary school math textbooks nor experience have prepared most teachers for this sort of math teaching. Fortunately the second volume structures its suggestions around particular vignettes of classroom teaching which allow teachers to see more concretely what the Council envisions, and to begin to try to imagine themselves and their own students "doing" mathematics in new and quite different ways. This excerpt from a working draft of *Professional Standards for Teaching Mathematics* suggests the richness of the descriptions and the commentary:

A class of primary students has been working on the following problem:

If we make 49 sandwiches for our picnic, how many can each child have?

These students have informal ideas about division but have not yet learned conventional procedures for dividing. They also have begun to develop some understandings of fractions, connected to their ideas about division. The teacher has selected this problem because she knows that it is likely to elicit alternative representations and solution strategies, as well as different answers. It will also help the students to develop their ideas about division and fractions.

After they have worked for about 20 minutes, first alone and then in small groups, the teacher asks if the children are ready to discuss the problem in the whole group. Most, looking up when she asks, nod. She asks who would like to begin.

The teacher provides and structures the use of time so that the children have opportunities to develop their solutions independently, with a few others, and then in the whole group. By asking who would like to share their solution, she encourages the students to take intellectual risks. She shows that she respects their work by consulting them before she moves to the whole group discussion.

Two girls go to the overhead projector. They write:

$$\begin{array}{r} 49 \\ \underline{-28} \\ 22 \end{array}$$

One explains, "There are 28 kids in our class and so if we pass out one sandwich to each child, we will have 22 sandwiches left and that's not enough for each of us, so there'll be leftovers."

Students expect to have to justify their solutions, not just to give answers.

The teacher and students are quiet for a moment, thinking about this. Then the teacher looks over the group and asks if anyone has a comment or a question about the solution.

The teacher solicits other students' comments about the girls' solution, instead of labelling it right or wrong herself. She expects the students, as members of a learning community, to decide if an idea makes sense mathematically. She respects and is interested in their solutions, but this does not mean that "anything goes." The next step is to verify the girls answer.

One boy says that he thinks their solution makes sense, but that he thinks that "9 minus 8 is 1, not 2, so it should be 21, not 22." The two girls ponder this for a moment. The class is quiet. Then one says, "We revise that. 9 minus 8 is 1."

Students respectfully question one another's ideas. The girls "revise" their solution because they have been convinced by the boy's point. There is no sense here that being wrong is shameful. The teacher, listening closely, lets the students talk; she does not get into the middle of their interchange.

Another child remarks that he had the same solution as they did – one sandwich.

"Frankie?" asks the teacher, after pausing for a moment to look over the students. Frankie announces, "I think we can give each child *more* than one sandwich. Look!" and proceeds to draw 21 rectangles on the chalkboard. "These are the leftover sandwiches," he explains. "I can cut 14 of them in half and that will give us 28 half-sandwiches, so everyone can get another half."

Listening to a variety of solutions, the students work together to solve the problem.

Students expect to have to justify their ideas.

"I agree with Frankie," says another child.

"Each child can have $1\frac{1}{2}$ sandwiches."

"Do you have any leftovers?" asks the teacher.

"There are still 7 sandwiches left over," says Frankie.

"What do the rest of you think about that?" inquires the teacher.

The teacher expects students to reason mathematically.

Several children give explanations in support of Frankie's solution.

"I think that does make sense," says one girl, but I had another solution. I think the answer is $1 + \frac{1}{2} + \frac{1}{4}$."

Students seem willing to take risks by bringing up different ideas.

"I don't understand," the teacher says. "Could you show what you mean?"

The teacher expects students to clarify and justify their ideas.

How does a math teacher begin to think about the changes that the Council is urging upon her? How do we prepare new teachers to think about teaching math in ways quite different from the ones that they experienced for many years as students? Teachers in professional development schools are grappling with these ideas, and their efforts may provide some help and inspiration for other Michigan educators who want to begin to make changes in their ways of thinking about "doing mathematics." ■

National Council of Teachers of Mathematics (1989) Curriculum and Evaluation Standards for School Mathematics, Reston, Va: NCTM.

National Council of Teachers of Mathematics (1989) Professional Standards for Teaching Mathematics: Working Draft, Reston Va: NCTM (This working draft can be ordered directly from the Council [1906 Association Drive, Reston, VA 22091] at no cost until the final draft is printed. *The Curriculum and Evaluation Standards* can be ordered for $25.00 a copy.)

A Community of Young Mathematicians

In 1980, when she began to have doubts about her mathematics teaching, Deborah Ball had been teaching at Spartan Village Elementary School in East Lansing for five years. The crisis arose when she moved from third to fifth grade. She had always taught math pretty conventionally: "I thought *I* should present it, and I should present it in ways that were both engaging to kids and conceptual, and that the math just wasn't that difficult to understand." Her students got right answers on their math papers, and so she assumed that the processes made sense to them. But in fifth grade the math was more complicated, and although she felt she was explaining new ideas with the same clarity, students did not get right answers with the same regularity. When she talked to them about their mistakes they seemed less clear on what they were doing and why. Troubled, she began to talk to Perry Lanier and Bruce Mitchell, at the University. They steered her toward books and articles about mathematics, about what was wrong with math teaching, and about how one might do it better. She began to rethink what it meant to learn mathematics, and to experiment with different ways of teaching. She also enrolled in college math courses – in algebra, calculus, and number theory.

From the courses she took and from the articles she read, Deborah learned more about the discipline of mathematics, and about alternate ways to think about teaching it. The Comprehensive School Mathematics Program (CSMP), which she worked with in her fifth grade, also helped. She had already started a master's program in reading at Michigan State's College of Education, and as her interest in the math grew, she changed her focus from reading to mathematics. In 1988 she completed a Ph.D. and became an assistant professor in the MSU Department of Teacher Education. She has continued, however, to teach math at Spartan Village Elementary School.

Students propose, argue, concur, dispute, and theorize.

Math as a Conversation

In 1990 Deborah's third-grade mathematics class is a community of mathematicians. Students propose, argue, concur, dispute, and theorize. Deborah explains that it is her view of mathematics, and her ongoing observations of these eight-year-olds, which shapes what she tries to do there. She gives over much of the agenda to the children's thoughts and comments, because to some extent she models her mathematics classroom on a vison of the community of adult mathematicians: she wants her students to see the enterprise of mathematics as a conversation, an ongoing community inquiry. But Ball is a teacher as much as she is a student of mathematics, and, she says, "There's a back and forth between reading about the work of mathematicians and watching and listening to eight-year-olds and trying to figure out what makes sense." She points to a couple of concrete ways in which her classroom differs from the communities of adult mathematicians: as a teacher she is concerned about the participation of all individuals – she must worry and take steps if some students do not understand the mathematics others are discussing; she also tries to keep competitiveness out of her third grade, even though it certainly has a place in the universities where mathematicians do their work.

Deborah's work both embodies and inspires the hopes for professional development schools. Every day she works with students, both in elementary school and in the university, trying to understand and extend their thinking. Her work with eight-year-olds informs her research; her reflections on her own practice, on the work of others, and on her academic discipline inform her work with children. And she continues to ask questions.

Visiting her classroom on a cold afternoon in late January, visitors from the University get a chance to see Deborah and her students at work on one of her newest interests: what can the idea of mathematical proof look like in a community of eight-year-old mathematicians?

Although math period has not yet begun, these third graders clearly feel that they are in the middle of something. As they trickle back into the classroom from lunch, still laughing and talking, they reach into their desks for their math notebooks, open them up, and inspect yesterday's work – all without prompting.

"Conjecture" Is an Everyday Word

"Are you guys ready?" asks Deborah. "I'd like to put up the conjectures from yesterday." "Conjecture" seems to be an everyday word for these eight-year-olds. Taped to the chalkboard are three pieces of orange construction paper, each stating a conjecture:

> *Lindiwe's conjecture* : No matter how many even numbers you add together, the answer will always be even.
>
> *Betsy's conjecture*: If you add an odd number of odd numbers, the sum will always be an odd number. If you use an even number of odd numbers, you will get an even number.
>
> *Keith's conjecture* : If you add an odd number to an even number, the answer will always be an odd number.
>
> *Sheena's Question*: Would it work with subtraction? Odd minus even equals odd?

(The variety of names on the papers around the classroom suggest the diversity of the students: more that half were born outside the United States; many learned English in this elementary school.) "Numbers that end in zero *may* always be even," volunteers

"Sean Numbers": Eight-Year-Olds Developing Number Theory

About a week and a half earlier, during a discussion of the properties of zero, a student named Sean had proposed that 6 is "both even *and* odd" because when six objects are grouped by twos, there is an odd number of groups. Neither his classmates nor his teacher agreed. Ofala argued that their working definition of an even number said only that when you grouped by twos, nothing would be left over – it said nothing about the *number* of groups of two. Deborah asked Sean "why it would be useful to have definitions such that some numbers would end up both even and odd?" (Sean responded that it wasn't necessarily useful, but that he was just thinking that it *could* be.) And Mei pointed out with an edge of exasperation that if six was "both even and odd," then so was 10. Sean considered this argument calmly and then agreed, with gracious formality. "I didn't think of it that way. Thank you for bringing that up. I see 10 can be an odd and an even number."

Reflecting on this discussion after the end of the teaching day, Deborah wrote in her journal:

> **(I'm wondering if I should introduce the idea that Sean has identified (discovered?) a new category of numbers – those that have this property he has noted. Maybe they could be named something. Or maybe this is silly – will just confuse kids since it's nonstandard knowledge – i.e., not part of the wider mathematical community's shared knowledge. I have to think about this.)**
>
> **(It has the potential to enhance what kids are thinking about "definition" and its role, nature, purpose in mathematical activity and discourse, which, after all, has been the substantive point of spending so much time on this this week.) What should a definition do? Why is it needed?**
>
> **For example, a definition of even numbers that says it's every other (whole) number starting with 0 (or 2) is pretty useless for dealing with Lucy's example of 1,421. Even the grouping by 2 definition (corollary to Ofala's odd number definition) is not too helpful for 1,421. But a definition *centered* on divisibility is (they all *relate* to this, of course, to a greater or lesser degree) because then you can show that 1,421 is odd because 1,000 is even, 400 is even, 20 is even, but 1 is odd. That leads to the**

Continued on page 7.

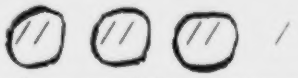
one child, carefully emphasizing the "may." Deborah writes this on the chalkboard. Tory reports that she is trying to figure out what happens when two "Sean numbers" are added together. ("Sean numbers" are even numbers that result from doubling an odd number. Their history – they were identified by a member of this class two weeks ago – suggests a bit about the way Deborah and her students approach mathematics. See "Sean Numbers: Eight-year-olds Developing Number Theory," in this issue) A third student reports that he is investigating to see whether the sum of an odd number and an even number is always an odd number; a fourth is looking at the results of adding two even numbers.

As these students have proposed conjectures, classmates have been volunteering to assist their investigations. Everyone now seems to know what they want to work on, but before they adjourn to their groups Deborah asks the class, "What did we prove yesterday?"

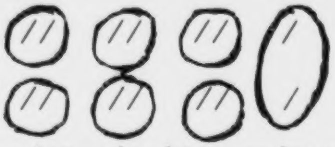
"An odd plus an odd equals an even," replies Jeannie. Temba comes to the board, writes

$$\begin{array}{r} 7 \\ +\,7 \\ \hline 14 \end{array}$$

and explains, pointing to the numbers, "Seven is odd, 7 is odd, and 14 is even." Definitely an example, but not, he agrees, a proof. Betsy offers a proof: "If you group an odd number by twos [she demonstrates by making seven lines, and circling three pairs],



you have one left over. You can make a pair with the two left over ones."



"I agree that this is true, but it wasn't the way we did it yesterday," Mei demurs. Yesterday, she asserts, they had defined an odd number as "an even number *minus* one." The two girls discuss whether this difference is important.

Mei argues that we cannot be sure that this statement holds true for really big numbers – "numbers in the thousands, or numbers that we can't say," because we haven't examined them. Jeannie disagrees. "*All* odd numbers have one left over. And you'd die before you counted every number. Even mathematicians can't count them all!" Arms wave. The third graders debate whether it is possible to *prove* anything about numbers that are too big to count.

Students Investigate

With no need for an organizational intermission, groups convene quickly at tables and chalkboards. Students make calculations in their notebooks and on the board; the few voices that rise above the busy hum are discussing math: "Yes, but does it *always* work?" "But that's a Sean number . . ." Two girls are investigating operations with Sean numbers by adding 22 and 22, making

"Sean Numbers" continued from page 6.

proof of Lucy's conjecture that "you have to look at the last number." (Many adults know this rule but do not know why it works.)

A second aspect of definition is that it facilitates discourse. That was where *we* started, because people were meaning different things by "even numbers" and that was going to make debates over the four conjectures difficult – *impossible*.

Another, I think, is logical partitioning? Well, in a case like this, maybe, but certainly not always, or there wouldn't be intersecting sets in number theory (e.g., prime numbers, odd numbers). That's where Sean's discovery fits, maybe.

Deborah turned the matter over in her mind over the weekend. On Tuesday, she wrote:

I decided to let Sean's idea of numbers that can be both even and odd have legitimacy by pointing out that he had *invented* another kind of number that we hadn't known before and suggesting that we call them "Sean numbers." He seemed quite pleased, the others interested. I said that when mathematicians invent new ideas or make new conjectures, sometimes their ideas are named after them and, after all, we had "Ofala's conjecture," "Nathan's conjecture," etc.

Commenting on her decision later, Deborah observed:

You could have interpreted this very differently – you could have simply seen Sean as a kid who was confused about even and odd numbers and just looked for a way to correct him. This is a very clear case of my view of mathematics shaping what I did. An observer who focused on children's feelings and self-esteem would probably have said, 'Here is a kid who is having trouble at school, and here is a teacher who is finding a way to make him feel good about himself.' But I at least as much saw it as an occasion for [the class] to see something about how these ideas are invented by people.

Students in Deborah Ball's math class work together on a problem.

a diagram which shows them the number of pairs in 44. As Deborah notes in her journal, they have some trouble deciding whether 44 is or is not a Sean number. When they finally agree that it is not, they conjecture that a Sean number plus a Sean number will always equal an even number that is not a Sean number.

Nearby a boy is drawing lines on the black board with colored chalk. A girl asks with indignant incredulity, "Are you *playing*?" Deborah moves from group to group, finding out exactly what every one is doing, and how they are thinking about the problem they have set themselves. A boy explains to her, "We only went up to 12,000. That doesn't prove it: what about all the other numbers?"

After 20 minutes of group work, Deborah brings the class back together and asks for a report on the conjecture that "an odd plus an even equals an odd." Mark explains that, "At first, we were getting lots of answers, but we weren't getting proof." Then, he continues, Betsy joined them, "and she had proof." When Deborah invites him to show one of his examples, Mark defers to Nathan, whose examples are "better." This turns out to mean that they involve larger numbers.

After consulting his notebook, Nathan writes on the board

$$\begin{array}{rl} & 100000000000001 \\ + & \underline{98124\qquad\qquad\quad} \\ & 98125 \end{array}$$

explaining as he does so that "Betsy said that when you add a short number to a long one, you have to match up the last numbers, and the next to last." He adds that since Lucy says that you only have to look at the last number to tell whether it is even or odd, he concludes that this number is odd.

Ignoring the gasp from the back of the room which greeted Nathan's addition, Deborah lets Betsy explain her "proof" that an odd number plus and even number will always equal an odd number.

Later that afternoon, Deborah reflects in her teaching journal on her decision not to correct the addition.

> **I did not comment, even though it was obviously wrong. At this particular point, we were concentrating on the issue of whether or not odd plus even equals odd, and, within that, the function of *examples* and other kinds of argument in proving that something was *always* true. In addition, getting them to understand what it means (or even what to do) to add these numbers is not a trivial matter . . . Nathan had followed a rule – that Betsy had said that**

> **when you add a big number to a smaller number, you have to add the ones first (or add the ones together?) – and my perspective was, what good would it do to say another rule? . . .**

In fact there is little danger that Nathan's error will go unnoticed. Just before the class ends, Jeannie raises her hand to disagree with the addition, and Betsy moves to correct the computation.

Accompanying the visitors to the car, Deborah remarks that "eight-year-olds really get hooked on number theory. They love it. It's hard to get them off it, to move on to other things." She explains that the unit on odd and even numbers began a few weeks earlier, out of questions that arose from the following problem: "If pretzels cost 7 cents and gum costs 2 cents, what can Mark buy if he wants to spend exactly 30 cents?" (See "Meanwhile at Elliott . . ." in this issue) Working on this problem, students saw some patterns involving even and odd numbers. Their questions have led them in a variety of directions.

As eight-year-olds discuss ideas about numbers, measurement, shapes, and probability, Deborah studies their learning and her own teaching and carries her thoughts and observations to undergraduate and graduate students at MSU, to her colleagues at Spartan Village Elementary and other professional development schools, and, through articles in professional journals, to mathematicians and teacher educators at other universities. Some students and colleagues visit her classroom. Others watch videotapes of her teaching and discuss them in courses or study groups. Used in different ways by different people, Deborah's classroom and her videotapes catalyze important thinking about teaching, learning, and mathematics. ■

Sharing a Classroom

When teachers join the Professional Development School effort, they open the door of their classrooms to outsiders – people from the University and the world beyond. What does this feel like? What sacrifices must they make? Do the demands and intrusions overwhelm any benefits they and their students may get from the association?

Sylvia Rundquist began her collaboration with Deborah Ball two years ago, when Deborah asked Jessie Fry, the principal of Spartan Village Elementary, to help her make arrangements to teach mathematics daily in someone else's classroom. At that time Sylvia had been teaching elementary school for two years. She did not feel very confident about her math teaching and the opportunity to learn from watching someone else attracted her. She volunteered.

Reflecting on the arrangement this spring, Sylvia reports that she has loved observing her students regularly in a situation where she did not have to teach: through the dual perspective of teacher and observer she has come to know the children better than she did when she was their only teacher. And she notices that at parent conferences she has more insightful things to say about them.

Just as she hoped, she has also learned a lot about the teaching of mathematics. Her earlier assumptions about math teaching, she re-

Continued on page 10.

The View from the University

When college teachers try to introduce new ideas about mathematics to the young adults in their classrooms, they get a variety of reactions. Those reactions reflect the fact that for the past 13 years these students have been constructing ideas about what teaching does and doesn't mean as they watched their own teachers assign workbook pages, write on the blackboard, guide discussions, and collect milk money. Because many of these college students will be the teachers of tomorrow, it is interesting to listen in as they wrestle with their mixed feelings.

Margery Osborne, a doctoral student at Michigan State University's College of Education, works on a research project which involves her in filming Deborah Ball's third grade math class several times a month. She also teaches a section of "Exploring Teaching" (TE-101), a course designed to help sophomores to examine their assumptions about teaching, learning, and schools. In late January, Margery's TE-101 students watched a video of one of the classes in which Deborah and her class investigated the behavior of even and odd numbers and the properties of zero. Margery then broke her class into groups to think about the similarities and differences between Deborah's teaching and that recommended by Good, Grouws, and Ebmeier in *Active Mathematics Teaching*. Today the groups are reporting their ideas back to the rest of the class.

Similarities and Differences

The reporter for the first group relates what she sees Deborah doing to the teaching of Vivian Paley[1], whose book about her kindergarten class students have read earlier in the term. Most TE-101 students like Paley's informal, conversational kindergarten, feeling that five-year-olds will learn social skills and gain self-esteem from exploring their ideas with a teacher who seems never to contradict them. Most doubt, however, that you could learn the information and academic skills that older children need without much more adult guidance. "There were times when we felt they went off the track and hit a blind spot. We felt she could give them more direction, lead them to something, because they were just spending a lot of time discussing zero."

She turns then, with apparent relief, to the more familiar approach of Good and his colleagues, who direct teachers to teach algorithms directly and briskly, assigning independent seatwork after 10 minutes of explanation and 10 minutes of oral questioning or "controlled practice": "Whereas in the book they had a step-by-step procedure in teaching math: the teacher explains and demonstrates, and gives them work and checks it."

Teaching Unsettling

The routines of Good, Grouws, and Ebmeier recall the third grades where these sophomores studied arithmetic 11 years ago. Deborah's teaching strikes a discordant note and raises some unsettling feelings. Janine fumblingly defends her own teachers against the unspoken criticisms of this new approach.

> **Talking about math reminds me that in elementary school, and in high school, and even here at State last year, I had math teachers who have said, "You're not going to understand this, you don't know why this happens, I just want you to do it." And it always bothered me, but I look at this now and I want to say "I love what Ball is doing. The kids are disagreeing and they're talking!" But I did learn a lot of math growing up, and I didn't understand it. But I did it. And I know how to do it now. Maybe I don't understand it, but I'm kind of educated because I do know that math. And I wouldn't have understood the principles. I still don't.**

Janine pauses and Margery tries to help her clarify her point. "So what you are saying is, why understand math?"

Janine nods, "I see what she is doing and I think that's great, but I just don't see how . . . " The last sentence dangles as she tries to

Sharing a Classroom continued from page 9.

calls, mirrored those of her own elementary school teachers: they seemed to believe – incorrectly, she now thinks – that "the more calculations I did, the more I would come to understand." As she discusses the ways her ideas have evolved through the last two years, she jots down five other things she has learned:

- **how much discussion there can be about math**
- **finding answers in different ways is OK – [children] don't all have to get answers in the same way**
- **that math is not all right and wrong – some issues are not resolved**
- **a great deal of day-to-day math involves estimating**
- **using concrete objects and manipulatives to find solutions is OK beyond lower [elementary school grades]**

Like outsiders who observe Deborah's math classes, Sylvia is often amazed at the problems her students can solve, and the level of their thinking. As she watches her third graders attack difficult problems with energy and sophistication, she raises the expectations she has for them in other areas. And she uses the vocabulary of the math class – estimating, predicting, making hypotheses – to extend her students' thinking across the curriculum.

But there are costs as well as benefits to her collaboration with Deborah, Sylvia admits. She and her students are locked into having math at a particular time each day, so that even if they become engrossed in another project they must turn their attention towards math at five minutes before one every Tuesday, Wednesday, Thursday, and Friday.

And there are some benefits which, the listener guesses, probably did not come without pain: Sylvia says that through her association with Deborah she has learned to be more flexible. "I was very rigid about my teaching." She felt possessive about her students and her classroom. Now Deborah's name is next to Sylvia's on the classroom door, and they do parent conferences together and cosign all communications that go home to parents. Because she shares her classroom with Deborah (and Deborah's entourage -- often a film crew as well as visitors from other schools and universities) she has to endure intrusions and rearrange her own schedule frequently. She is managing to do all this, and even to feel increasingly relaxed about the changes. She is also learning, she says, that she can speak up and set some limits: After a bit of silent suffering she decided that it really would be all right to ask the visitors *not* to sit on her desk.

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[1] Paley, V. (1981). *Wally's Stories.* Cambridge: Harvard University Press.

make sense of her thoughts, memories, and feelings. "Maybe with something else, but with math?" Her voice trails off uncertainly.

A classmate dissents:

> The problem with that is that . . . once you stop using what you were just told to do, you forget it . . . I mean, you don't really learn it. And people say, 'Well, I'll never use math. But there are a lot of concepts that, if we had learned them so we understood them, maybe we would use them.

To most of these college sophomores, math is a long list of procedures and algorithms. They wonder if it is possible to spend an hour or more struggling with one problem and still "cover" the curriculum. But even if it were, the spokesman for the next group reports, many would prefer to see the teacher run the discussion with a firmer hand:

> [Good, Grouws, and Ebmeier] suggested using the first 4-5 minutes for review. Then you could let them discuss for 20 minutes, but not go on the whole hour. After a while it seemed like it was just the same four or five students . . . Plus, she never told students they were right, or gave them any encouragement and I think that that kind of leaves students hanging. I mean, I think you should tell kids the right answer after a certain point.

"What is the right answer?" asks Margery. Students' concern about the length and form of the third grade discussion has crowded out the mathematical content so completely that a puzzled silence falls. "About zero," Margery reminds them. No one seems to be sure that they really know, but neither do they pause to discuss the question – or to wonder at the interest that these eight-year-olds display in an abstract point of number theory. "Nothing," calls out one. "It's not a number."

"I think that when you are listening to other people present ideas, whether they are right or wrong, you are learning something."

But as some students identify the ways in which Deborah's teaching troubles them, others are beginning to look at their own dissatisfactions with her teaching more skeptically – to question their own questions. Judith volunteers:

> I think a lot of our problem is that a lot of us, probably all of us, we've been taught in the traditional way: "Here is what this is. Here's how we are going to do it. Go ahead and do the problem." And it's hard for us to try to understand: looking at this other form of teaching, we think, "How can kids do this? It's so hard for them." But maybe if they do it all along, maybe it will be better for them.

This student is worried, however, about confusions generated by different teachers' expectations: "What if they go to fourth grade and the teacher *doesn't* do this? And the kids are challenging and saying 'I disagree.' 'I agree'? And," she laughs, "the teacher tells them to shut up. You have to have consistency."

Alicia voices a different concern: she doubts that mathematics is logical enough to support the scrutiny that these eight-year-olds are giving it.

> They were learning the theory behind all this math, but I mean, there are so many exceptions anyway that at first they'll think, 'Oh, great,' but once she adds in, 'Oh, this isn't right: this is an exception,' ultimately they won't want to know everything behind it.

Re-examining Beliefs

In applauding the way that Deborah's students work together on a problem over a considerable period of time, a young man asserts, "The teacher can't just tell them the answer. I think it is good the way that they try to come up with it by themselves."

A groupmate disagrees:

> I don't think that's right. A lot of times the teacher just has to give you the answer and you have to accept it. There are lots of things in life you just accept. At least, I've been taught that way. . . . And, I mean, discussion with all the little friends, to me it seems useless. Because there are kids who never talk. I would never talk, whether I understood it or whether I didn't.

After a thoughtful silence, another student points out that she is assuming that those who do not join the discussion learn little:

> There's a problem in saying that if you aren't saying

anything, you aren't getting anything out of the discussion. That would mean that people in here who haven't said anything this period, that the whole hour has been a waste for them, and they haven't learned anything. But I don't think that's true. I think that when you are listening to other people present ideas, whether they are right or wrong . . . you are learning something. . . . A lot of times the people who don't speak learn the most, because they have the observer's point of view, and they get to see the whole of the discussion. Like right now I'm talking, so I'm not listening in the same way, figuring out what makes sense.

As the class continues to react to what they have liked and disliked, students raise questions and challenge one anothers' assumptions. Do you learn more from a teacher who comes on strong and displays more "personality"? Do you learn for the "wrong reasons" with such a teacher? Will Deborah's students, even if they do not outscore others on standardized tests in third grade[2], carry with them, into high school and college, the energetic enthusiasm for mathematics that they display in this class?

Talking about their responses to Deborah Ball's teaching, these students lay on the table some fundamental beliefs about teaching, learning, and mathematics. In the next few weeks they will get a chance to visit Deborah's class, to ask her questions about her teaching, and to re-examine, both in discussion and writing, the notions that have surfaced today. Some ideas will change; others will not. But nearly everyone will leave their introductory education class seeing some new possibilities and new complexities in teaching and learning. ■

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[2] In fact, Deborah's students do exceedingly well on these measures.

Meanwhile, at Elliott...

Like the sophomores in Margery Osborne's exploring teaching class, the five classroom teachers in the Math Study Group (MSG) at Elliott Elementary School in Holt remember the math classes of their childhood very well. Unlike many of the college students, they are sure that they want something different for their own students. All have been teaching for at least four years; none are satisfied by the way in which they have been working with their students on mathematics. Several joined the PDS effort specifically in order to look for ways to move beyond what they knew about math and math teaching.

In an effort to deepen understanding of the teaching and learning of math, the group (which includes, in addition to the five classroom teachers, two "interns" who graduated from college only last year, one graduate student and two professors from MSU) has read articles, interviewed students, shared teaching journals, worked together on math problems, discussed personal math anxieties, and analyzed curricula. Shortly before Christmas, the group watched a video of Deborah Ball teaching her class at Spartan Village about negative numbers. Impressed by the third graders' command of a language for discussing these abstract ideas (in the interviews, teachers had felt that *their* students seemed to lack a language for expressing their mathematical understandings), and curious about the strategies Deborah used to initiate students into this sort of discourse, the teachers welcomed Deborah's offer to come to Elliott and teach a lesson. Karen Dalton volunteered her third-grade classroom and the group arranged to videotape the lesson, watch the tape together, and discuss it with Deborah.

The film they view on February 12 shows Deborah giving the Elliott third graders a problem she has investigated with her class at Spartan Village three weeks earlier: "If pretzels cost 7cents each, and sticks of gum cost 2 cents each, what are the possible purchases a child who wanted to spend exactly 30 cents could make?" In Deborah's class, this problem has led to several weeks of interesting work on odd and even numbers – including that described in "A Community of Young Mathematicians" and "Sean Numbers" (in this issue of *Changing Minds*). After some discussion, the eight-year-olds break into groups and start experimenting with number combinations, talking animatedly about prices of gum and pretzels.

The voices are those of treasure hunters. As a boy in a blue sweater counts by two's ". . . 18, 20, 22 . . .," other children hiss excitedly, "I got one," "Three pretzels won't work," "Try two." Karen helps a group which has gotten confused enroute to a workable combination: "You had 2 pretzels and 7 pieces of gum and it was 28

The Elliott Math Study Group at work.

cents. How would you make it 2 cents higher?" . . . "Would that do it? Try it." Deborah crouches next to a pair who are trying to decide whether they can buy 7 pretzels.

After 15 minutes the class reconvenes to discuss their findings: a girl who proposes 4 pretzels and one piece of gum explains, with the help of several friends, how she figured out, by repeated addition, that 4 pretzels would cost 28 cents. Another youngster proposes that 15 pieces of gum will cost exactly 30 cents.

"Why," asks Deborah.

"Because," volunteers a classmate, "15 plus15 equals 30."

"But I don't see what that has to do with gum," probes Deborah. "I thought gum cost 2 cents?"

Another little boy explains why 15 plus15 is the same as 15 groups of 2.

Not all answers are on the mark: someone has forgotten that gum cost 2 cents and proposes that the purchaser can get 23 pieces of gum and one pretzel. But this error leads eventually to the interesting conclusion that it is *impossible* to buy *one* pretzel and come out even. As their 40-minute math period ends, Deborah is asking students to think about whether they have found all the possible purchases.

A Question of Equity

As the screen on the monitor goes dark, the Elliott teachers turn first to questions of equity: how does a teacher meet the needs of a diverse population of students? After some discussion about whether a chart might help some of the slower kids to see the patterns involved, a teacher puts the question more generally: "That was one of my questions, watching the variety of kids in there: How do you keep the kids who have already got all your solutions . . . challenged, and still going; and *then* how do you get those slow kids to come along?"

In responding Deborah turns first to the lesson they have just watched, and then to the more general curricular strategies that she uses to address diversity.

> **Well, it didn't happen in her class that anyone got all the solutions. [Two boys] thought they had found them all, and I asked them if they were sure and they said they were, but in fact they hadn't. That was one of the ways I used the group discussion – to help them see that they didn't get all the solutions, so that next time they get such a problem they'll try to push a little harder, to prove to themselves that they are done.**
>
> **I guess part of it, to me, lies in trying to figure out problems that are rich enough that it's not so likely that some people are going to be totally done and other people are going to be totally stuck – that there are just different things that you could think about or do with the problem. And then, I usually try to have thought through different kinds of extensions that kind of relate to the problem.**
>
> **Part of my goal would be that by this time of year, in my own classroom, I would want kids to have questions that the data raised for *them*. So partly I would be trying to get them to a place where they start raising questions for themselves, saying "Hmmm,**

> **I'm noticing something here." or "Hmmm. I see a pattern."**
>
> **And *at this point*, in my own classroom, there's some sign that they do do that. Whereas earlier in the year I'm showing them that that's one thing that you do, when you think you are done. You say, "Am I sure this makes sense? Am I sure I'm done? And is there anything interesting going on here that goes beyond this problem?" I try to get them used to the idea that that's part of what you do when you do mathematics. Traditionally when a teacher gives you a problem, you do it, and then you are done and you just wait until she tells you what to do next.**

A few minutes later a teacher who taught some of these students when they were younger makes an observation which goes to the heart of the equity question: "The kids who were contributing the most were not the math stars." "No," the third grade teacher agrees, "those kids held back. They were afraid to take risks with a kind of math they weren't used to." She had noticed one of her best students whispering ideas to the boy in front of him, and waiting to see how Deborah would receive his second-hand suggestions. "This is really important," emphasizes the first teacher. We have a lot of needy kids. The tape is evidence that this teaching would work for them."

Members of the Study Group comment that students respond positively because Deborah accepts all comments and suggestions. "But it's a particular version of accepting everything," Deborah reminds them, "because it's accepting everything but also expecting that everything has to be backed up." It is, she explains, difficult to respond to all comments with the same neutral interest.

> **I think the hardest thing for me has been to learn to act neutral when kids are right. Most of us are pretty inclined to be at least *nice* to kids when they are wrong, so it's not such a big leap to learn that if something's wrong you are going to say "why?" and sound kind of neutral about it. But what's harder is when it's right, because we are so much in the habit of saying "good job." But it's so key to do it both ways. If you only ask "why?" when they are wrong, then you have just gone to a new system of cuing them.**

"I guess part of it, to me, lies in trying to figure out problems that are rich enough that it's not so likely that some people are going to be totally done and other people are going to be totally stuck."

How to Justify

Like the college sophomores, the group wonders how Deborah justifies to parents, colleagues, and administrators spending so much time on a few problems. She explains that she shows people who ask this question the district objectives in mathematics for the grade she is teaching. "And most of the problems that my kids are doing address a surprisingly large range of these objectives." The unit on even and odd numbers that she has just finished, for example, included work on more than half of East Lansing's conceptual and computational objectives for third-grade math.

Instead of seeing the curriculum as "little bites that come one right after another," Deborah uses the model of a sandwich. With a good problem "you are working on a bite that is kind of deep and has a lot of layers you want to get at." And although her students do not use flashcards or practice adding or subtracting in isolation, they do extremely well on tests that require knowledge of number facts because they do many computations as they probe a problem or conjecture.

New Vocabularies: Conjecture and Revision

Everyone wonders how Deborah helps her students make the transition from a traditional mathematics classroom to one in which all answers are considered and discussed. Deborah explains that she introduces new words like "conjecture" and "revision" as the need arises. The new vocabulary, she thinks, helps to build a mathematical community; it also draws attention to activities that she values – like proposing "conjectures" and "revising" answers in light of points made by others. In addition, she coaches students in behavior that contributes to good conversation. When they come to the board, for example, she tells them to stand to the side of their diagram and face the class as they explain their reasoning. She sets up explicit rules about responding to the comments of others: no one is to interrupt, because students need to think as they talk, and to be able to revise ideas as they propose them; when your classmate finishes what he has to say you may (1) ask for clarification or repetition; (2) agree or disagree, giving reasons; or (3) explain how you got the same answer by a different process.

New classes seem to pick all this up easily in the fall, but students who enter later in the year need more explicit guidance. She tells of a newcomer who laughed at someone whose English was limited. Upset, she stopped the class to discuss the incident. "Why did I get upset?" she asked. "Because," one student answered, "if someone gets laughed at, then he won't want to contribute to the discussion." "And why would that matter?" asked Deborah. "Because if someone is too nervous to go to the board," explained the newcomer, "then he won't learn anything and he won't be able to show the teacher what he knows." Looking puzzled, another youngster raised his hand. "I don't think of it that way. I think if someone is afraid to go to the board, we *all* lose out. Because that might be the only person who has an idea that we all need in order to solve the problem." The eight-year-old had summarized elegantly Deborah's goals as a math teacher: a classroom in which a community of young mathematicians take one another's ideas seriously and work together to construct new understandings.

At three o'clock Deborah excuses herself apologetically, and heads for another meeting. As the door closes behind her, a group member sighs, "I just feel so inadequate, in terms of my subject matter knowledge." Another nods agreement: "I just can't imagine being able to be this thoughtful every day. Maybe one day a week . . ."

As experienced teachers who were already questioning their own teaching and asking hard questions about their students' understanding of mathematics, the Elliott Study Group came to Deborah's lesson with different concerns than Margery Osborne's TE-101 students. They believed that what Deborah was doing was difficult and, as someone had said in the December meeting, "Not the kind of thing you can be 'trained' to do in a one or two day inservice." They were certain that this sort of teaching required a lot of mathematical knowledge and a different and more demanding sort of planning. They seem to understand immediately what Deborah means when she talks about achieving many objectives with the same problem. Because they know their students, they can see how Deborah's teaching really does address questions of equity. They sense where the discussion is going, and why Deborah needs to respond to students' contributions with low-key, neutral acceptance. They bring to the video an appreciation of children and curriculum that the TE-101 students are just beginning to go out in search of. ■

"This Teaching is *So* Hard": A Teacher Learns on Her Feet

As interest in teaching for understanding has grown over the last few years, teacher educators have started to ask what skills, resources, and knowledge experienced teachers would need to learn in order to teach mathematics in a way that has students reasoning rather than memorizing rules. Like Deborah Ball, Maggie Lampert has been developing this kind of math teaching at Spartan Village School. During her first year of graduate study at MSU, Ruth Heaton, who had taught elementary school for nine years but had no special background in mathematics, became interested in the work that Maggie and Deborah were doing. In the winter of 1989, she and Maggie began to talk about the possibility of Ruth's taking over the math teaching in another class at Spartan Village so that Ruth could learn more about teaching for understanding and she and Maggie together could study what would be involved in Ruth's changing her practice. Since Ruth aims to become a teacher educator herself, she hoped to learn valuable lessons from this experience.

In September Ruth began to teach math in the fourth-grade classroom of Jackie Frese. Every Wednesday and Thursday until Christmas, Maggie watched Ruth's class, made notes in a journal she kept on Ruth's teaching, and talked to Ruth about what she had seen; Jim Reineke, a graduate student at the College of Education, studied her learning and filmed her class on Monday and Wednesday.[1]

Clearly, Ruth doesn't have much opportunity to make mistakes in private. On September 27, after she had been working on this teaching less than three weeks, she wrote in her teaching journal:

> **This teaching is so hard. Maybe [another professor at the College] was right. It would have been better to have some time to mess around on my own before having people observe. As he said, you need some time to make an ass of yourself. Well, I have. It really is hard – one thing that is hard is having things pointed out to me that I already recognize. It's also difficult to have things pointed out that I don't necessarily recognize. I am taking enormous risks.**

Not having realized that "whole numbers" would puzzle the children, she had not thought about how she would explain the term.

What Was So Hard?

Most of the time, however, Ruth talks cheerfully about her struggles – for example, she shared them with the faculty at Spartan Village in November. And in February she volunteered to watch a video of the September 27 lesson and talk about what was "*so* hard" as she first began to change her math teaching.

As the TV screen lights up, we see the math problem printed on the chalkboard:

"What whole numbers could be put in the boxes?"

$$26 - \square = \square$$

A small hand shoots up: "What is a whole number?"

"Like one, or two. Not $1\frac{1}{2}$," explains a classmate.

"Not $1\frac{3}{4}$" adds another. Quite a few children volunteer examples of whole numbers:

"A million."

"Zero."

"That's not a number," a girl objects.

"What is it?" Ruth asks.

"It is nothing," responds the 10-year old. After a brief but spirited discussion of zero (these students had Deborah Ball for math last year), students return to naming large whole numbers.

"101."

"60,000."

"Is negative 13 a whole number?" someone asks.

"Yes," answers Ruth. She calls on another child with a raised hand.

"I don't understand."

Ruth stops the VCR and turns to me, laughing: "This sort of thing happened a lot in the beginning. I wouldn't anticipate problems with words." Not having realized that "whole numbers" would puzzle the children, she had not thought about how she would explain the term, or help the children to investigate its meaning. As Ruth switches the tape back on we see her gesturing toward the line of numbers running across the top of the chalkboard, saying "Whole numbers are the ones that are on the number line." Mathematically, this is not precisely correct, as Maggie, who was observing that day, explained in a journal she writes on the lessons she observes.

> **[T]he *numerals* written below the red dots on the number line are what are being referred to as "whole numbers" i.e. $6\frac{1}{2}$ is certainly on the number *line* although the numeral "$6\frac{1}{2}$" is not written on the *chart* that represents the "number line." A complicated and abstract point which reminds us of how much mathematics is involved even in the problems that elementary schoolers undertake.**

Issues Identified

Maggie uses the journal to help Ruth identify mathematical and pedagogical issues:

> **It's hard to always know how to handle these ambiguities, even when you are aware of them, but the issue of how to use the number line in math teaching is a complicated one. Within *mathematics* the importance of the number *line* is that it represents *continuity* . . . that is, it represents the idea that there are always more numbers *in between* the other numbers.**

By now the children have opened their math notebooks and begun generating number pairs for the open sentence on the board. The screen shows Ruth crouching next to a child who has just recently joined the class and speaks no English. Her head on a level with his, she gives him an example of two numbers which, when inserted in the boxes, will complete the equation. Probably this is the first time that this little boy has encountered a problem of this sort. Nearby, another child explains whole numbers to the little girl on her left: "All the numbers that aren't like $1\frac{1}{2}$, $1\frac{1}{4}$ . . ."

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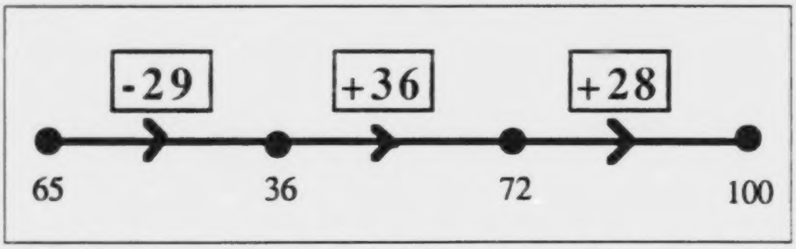
[1] Jackie Frese took over the math teaching during winter quarter while Ruth taught at MSU. In late March Ruth returned to Spartan Village where she will continue to teach fourth grade math for the remainder of the year.

Ruth talks quietly with a youngster who is trying numbers with nine or more zeros, and then leaves him with a manageable challenge: "Raise your hand when you have five reasonable ones." Another boy has generated quite a few appropriate pairs and tells Ruth that they did this last year. She gives him a somewhat more difficult problem to work on: 26 - ____ < 10. He does not, however, pick up on this challenge, and Maggie, in her journal note, helps Ruth to think about what may be going on inside of his head.

> **[H]is behavior makes me think of how much *content* and social/emotional issues are intertwined.**
>
> 1. **It seems to have been hard for him to cope with having been *singled out* at his table. Right after you walked away, the boy across from him (who had been listening) joked a bit with him and then they all took up fooling around with rulers.**
>
> 2. **It may have been that he felt "I've done my job here, why should I be 'rewarded' by being asked to do *extra* things, when these other guys at my table haven't got nearly as many possibilities down in their notebooks as I have in mine.**
>
> **These two issues are related to the problem of multiple levels of ability and trying to develop a classroom culture that supports everyone working at his/her own level. In certain social groups (maybe the one [this student] is in) it is probably not very popular to be really good at a school thing, like math: not to show up your buddies.**

A little girl consults with Ruth and then carries her math book over to the number line posted above the blackboard. After a few more minutes, Ruth calls the group together, congratulating them on the way in which they have worked on the problem, while noting that a few did not stick to math and "that's not okay." Then, before we can blink, teacher and students are off on a second, quite different, problem: they are constructing an "arrow road."



An Arrow Road. Ruth put the part in black on the chalkboard. Students added the numbers in pink to show the operations which would get them from one number to the next.

Struggle with the Text

Ruth turns from the screen, shaking her head in disbelief. Why, she wonders aloud, had she marched onto a new problem without even discussing the answers the students had generated as they worked independently? Now, she says, she would spend the whole math period exploring what they had found and the sense they had made of their answers. She explains that as she puzzles about why it had, at that time, seemed reasonable to her to proceed in this way, she realizes that this was part of her struggle to figure out how to use the math textbook in her teaching: the book links the two problems together in one lesson. She laughs as she points out that she had ploughed ahead without ever finding out whether the students have even understood what they have done because that's the way the textbook had laid out the lesson. Showing me the teacher's edition, which provides a complete script for teachers' questions *and* students' answers, she observes with a chuckle, "When I ask 'why?' it throws everything off."

As Ruth and I turn back to the screen, we see the arrow road directing students to figure out what operation they will need to perform to get from 65 to 36. A little girl comes to the board, but as she explains her reasoning, she forgets to "borrow" in the tens column and a classmate gasps audibly. "If you are going to disagree," Ruth urges, "just raise your hands quietly." A boy corrects another student's slip of the tongue, and Ruth comments to me,

> **There's a lot of social stuff going on here, a lot of getting really picky, and they're not picking at the mathematics, they are picking at each other. It was really hard to know what to do about that. . . . There's a fine line. You want**

them to challenge each other, but what is it that they are challenging?

A boy has volunteered that in order to get from 36 to 72, he would add 36. Ruth asks him how he has arrived at this answer, but gets no substantial response.

"Did you just know it in your head?"

"Yes."

In this episode, Ruth comments, she has entirely failed to probe his thinking – something she would surely have done later in the term. "I think some of that reflects my own uncertainty at that point, because I look at this now and I think, 'Why didn't you push him?'"

The class presses on, considering how you decide whether to add or subtract in order to get from one number to another. One student explains that she had subtracted because if she added she "would get the wrong answer." Others describe their thinking more lucidly. One little girl gets 38 when she subtracts 72 from 100; when a classmate explains where he thinks she has gone wrong, she asks some questions, ponders his answers for 30 long seconds, then announces with a satisfied nod, "I agree."

As the class adjourns, Ruth shares with her students her puzzled concern about the nature of the challenges she is hearing.

> **I have to think more about this, but some of the challenging that was going on here today I thought was kind of picky. . . . More like you were challenging each other. And that's not what these discussions are about. It's not your chance to dig at one another, it's an opportunity to think about mathematics.**

Her tone is genuine, thoughtful, and puzzled.

Ideas Change Quickly

It has been hard, Ruth explains as she rewinds the tape, but she knew at the outset that she would not start out an expert. "The struggle was, in some ways, the point." The changes in her teaching have been large and important, she reports. "It was as hard at the end of the term as it was in the beginning, but different things were hard." By December she was worrying most about what sort of math problems she would give them. "And the math. That was hard all the way through, but probably it got harder as I recognized more what I didn't know."

Looking at tapes of her teaching, and reading old journal entries, was often painful, because her ideas were changing so quickly.

> **Doing this lesson now, I would have much more discussion. It seems like I was so controlled: I mean I'd put one little thing on the board and talk about it, and then another little thing. And although there were some places here where I pushed the kids to talk more, now I would do this *much* more. But at the time, that was really adventuresome. . . . One thing that strikes me when I watch it now is how serious and nervous I looked in the beginning of the class, walking around. That was all this stuff, like about whole numbers: I would think, '*What* are they going to come up with next?' And so, when I would go around and talk to people, I was really fearful of their questions.**

Ruth's apprenticeship has given everyone who has watched her new insights about what is hard for a teacher who tries to change her way of teaching math. Together she and Maggie are learning more about how to organize apprenticeships for teachers and graduate students like herself. They are considering how a teacher might begin to address limitations in her understanding of a subject she is trying to teach. Equally important, Ruth has shown colleagues in PDS and in the University that, with help and hard work, a teacher can move in a few months to fruitful and exciting classroom conversations about math and towards an approach to teaching which feels more satisfying. ■

Special Circumstances: Rethinking Middle School Math

> **I guess overall the biggest thing [about the Professional Development School Project] is that it has really made me think . . . . Whereas before, I just *did*. I didn't think about what I wanted them to do, and maybe why. You start asking kids questions. Previously, you didn't do that. You weren't thinking yourself, so why would you ask kids to think?**

Three years ago, Patti Wagner taught math to the seventh grade special education students at Holmes Middle School in Flint.

Math had never been her favorite subject, but this time she felt totally frustrated – "I was angry at myself, and angry with the students, because I knew they had not gained any understanding of math." Those who were having trouble with addition at the beginning of the year were still struggling in June. She had led them through the textbook, but neither she nor the students had enjoyed the journey. At the year's end she was glad to hand math instruction over to another member of the special education team.

In 1989 she agreed to give it another try. Hoping for inspiration, she sat in on the mathematics portion of the Educational Extension Service's Summer Institute: "I had never thought about teaching math differently. I had never put a whole lot of thought into math, period. Part of me always thought, 'Well its just sequential and it's very easy to teach: once you know a skill, you just build on it.'" She was excited by the introductory session in which Glenda Lappan gave her audience sets of one inch cubes and asked them to explore area and perimeter. "It was so interesting to see that people did it all different ways, and all of them were right. In math we've been trained to think that there is only one right answer." She also attended a session in which Maggie Lampert showed videos of her class at Spartan Village Elementary School and talked about the ways she incorporated writing into math instruction. She began to think differently about her own math teaching, and to talk to Sandy Wilcox, a faculty member at MSU who was working with the Holmes PDS team, about new materials and new approaches.

A Different Approach

This year she has taken a very different approach to math instruction, choosing problems that require thought rather than the simple application of an algorithm, using manipulatives for the first time, and encouraging students to talk about the ways in which they are making sense of math. She has learned a lot about their thinking: "It just blew me away," she recalls, "when I saw that my kids were coming to my class with such a lack of understanding." And although she speaks modestly about her accomplishments – "I'm just beginning. I have a long way to go" – she notes with satisfaction that now *all* of her students enjoy math. "And I *think* they are getting a better understanding, though it's difficult to measure."

Ms. Wagner finds that as she rethinks the *way* she teaches, she has to face basic questions about *what* to teach.

This year, instead of following a seventh-grade math textbook, she has focused on very simple and basic things – "Things you would think these kids would know already – like 'what does 3 plus 4 really mean? Now, write a story for that.' It was very difficult for them." Since September they have been working to understand computational processes that they have been encountering in school for the last six years. Last month they worked with one-inch tiles to find ways to represent multiplication visually; since then, they have been trying to make sense of division.

Today, Patti's small fifth-hour class is creating pictures to represent written division problems visually. After some cheerful warm-up conversation about [illegible], Patti hands each student a piece of paper and asks them to write a division problem on one side of it, and to draw a representation of the problem on the other side.

After about five minutes everyone is finished. Hands wave eagerly when Patti asks who wants to put a problem on the board. She chooses a girl in a blue sweater who comes to the front of the room and draws seven boxes, each with an apple inside. "Seven divided by seven," calls a classmate; everyone agrees.

The next volunteer, however, leads her classmates into more complex terrain. She puts three circles on the board, with five lines (representing five sticks of gum) in each one. "Five divided by three," announces the girl in the blue sweater. "Are we sure?" asks Patti. Four heads nod confidently. "Are there just five sticks of gum?" asks the teacher? The girl reconsiders and revises her answer: "15 divided by 3." "Why divided by three?" Patti wonders. "Because it would be easier," explains a student.

When they have sorted this out, and come to a more satisfactory reason for writing the problem as 15 divided by 3, Patti turns to the original answer: "Could we show 5 divided by 3? Arms wave eagerly; everyone wants to try. A tall boy draws three circles, with three sticks in each, but everyone else agrees that this shows 9 rather than 5 divided by 3. A second boy offers another suggestion: three circles, with one stick in the first and two in the second and third. The tall boy gazes thoughtfully at this sketch; eventually he nods, "Oh, I get it." Patti puts the question into words: "If there are 5 pieces of gum and 3 kids, how many will each kid get?" "Two."

"One." "Two will get two, and one will get one," explains someone, and everyone nods, apparently satisfied.

The problems are not easy for these seventh graders, but everyone seems to be highly engaged. A girl who entered the class looking tired and indifferent strains forward, waving her left hand eagerly as her classmate puts a picture on the board. She is drawn into the lively inquiry all around her. Some students look puzzled when their teacher disagrees with their answers, but no one withdraws from the conversation in discouragement, and no one wanders from the subject. A minute before the bell, Patti collects student papers and asks whether anyone watched the Grammy awards the previous night. It is the first time that hour that anyone has spoken of a world beyond mathematics.

New Problems Raised

Subjecting her math teaching to her own critical scrutiny has raised new problems, even though it has been satisfying in some ways. To begin with, Ms Wagner has found that as she rethinks the *way* she teaches, she has to face basic questions about *what* to teach:

> **I'm still very much struggling with what my kids need to know, as learning disabled students, and as people who in six years will be out on their own. I don't feel that months of trying to conquer long division will make them more prepared. I try to think about how I use math, and then ask, why would they need to know this? Sometimes I don't know. Yet for some who continue struggling with concepts and understanding, getting them to enjoy coming to math class is a beginning.**

She also finds herself wondering about common practices that she previously accepted matter-of-factly. "They really need to reach and reteach teachers," she comments, reflecting on a recent incident. A special education teacher she knew had been trying to help a learning-disabled student who was mainstreamed in math and was confused about a homework assignment involving addition of fractions.

> **Evidently the teacher wanted them to do it in a particular way. And evidently the student did not get this down. Well, [the special education teacher] taught her to do it the way she thought easier and clearer. The [regular] teacher marked them all wrong. The answers were right, but she wanted them to do it in a particular way.**
>
> **In the past I might have just said, "Well, you should have listened." But now I wonder, should this have happened? Should we be teaching children that there is only one way to get the answer?**

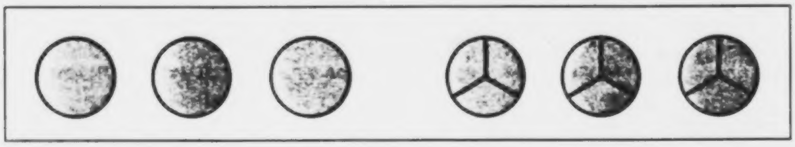
It is clear to teachers who are thinking about opening up their math teaching that they need to know more math to teach students who ask questions and try a variety of approaches and talk about the ways that they are understanding a math problem. It is also clear that teachers need to give math teaching a great deal of time and deep thought. Many elementary-certified teachers would argue persuasively that because they believe that the language arts (reading, listening, speaking, writing) are more important, they invest their time and energies there and simply do not have the time or training that innovative math teaching would take. Patti does not feel particularly confident about her math knowledge, and even after the exciting work she has been doing in math she still gives the bulk of her out-of-school time to language arts. Nonetheless, she has made major changes in her mathematics teaching, and her special education students seem to be understanding what they do and having a wonderful time. ■

Projects at Holt High School

When Sandy Callis Bethell started teaching at Holt High School in 1984, she was, she now recalls, "a very traditional, executive-style teacher." But when she returned to Michigan State's College of Education for graduate study three years later, she began a process of re-thinking schooling, learning, and mathematics.

On Friday, February 23, 1990, the visitor to Sandy's tenth-grade Practical Mathematics class sees little sign of the teacher Sandy says she used to be. Within minutes of the bell, the 22 boys and girls present, about half of them officially designated as having special needs, (learning disabilities, emotionally impairment, or both), have organized themselves into groups. Sandy hands each group a piece of paper bearing a number sentence involving two equivalent proportions ($\frac{1}{3} = \frac{3}{9}$; $3 = \frac{9}{3}$) telling students to work together to represent their sentence visually. A tall boy glances truculently at the paper Sandy has handed him. "How am I supposed to do this?" "That's the question," she smiles. "Then I'm not going to do it." He lays his pencil on the desk with emphasis. "You can do it," Sandy reassures him, without missing a beat. As she heads off to answer another question, he retrieves his pencil and reexamines the paper.

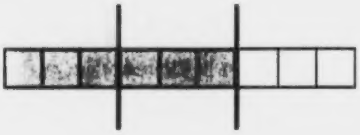
Jean Tomlinson, a special education teacher who team teaches with Sandy, moves along the other side of the classroom, offering suggestions and encouragement. As the groups finish, each one sends a delegate to the blackboard; after about 10 minutes there are seven numbered drawings on the board. Sandy directs the students to put the numbers on their papers and write next to each one the equation they believe the picture attempts to represent.



Ten minutes later the class goes over the answers together. Students immediately identify one drawing that is incorrectly made: it shows $\frac{6}{15}$ equal to $\frac{1}{4}$. Another diagram, which shows three shaded circles followed by three shaded circles divided into thirds, stumps everyone but the group that has created it (see figure above).

"Does anyone want to take a stab at it?" Sandy asks. "$\frac{3}{3} = \frac{1}{3}$?" ventures a brave soul. "$\frac{9}{4}$?" suggests another. Together they work through these hypotheses until they arrive at a solution that, after some discussion, makes sense to everyone: $3 = \frac{9}{3}$

The next two problems are more straightforward, showing identical rectangles with identical shaded portions, divided up into different numbers of pieces. But the fifth one provokes cries of outraged confusion:



"There's only one picture!" "It doesn't make any sense." No one has a correct guess, so a student from the group who generated the picture comes to the board and writes $\frac{2}{3} = \frac{6}{9}$. "A great picture," comments Sandy. "Why did you decide to do it that way?" "It was too simple," replies the tenth grader.

Sandy is pleased with this class, especially when she compares it with the basic math class she taught before she went to graduate school. She is struggling to develop a curriculum which engages the students with math concepts and not just with practical applications of arithmetic skills. The atmosphere is comfortable and cooperative: "They are responding very positively," says Sandy. "They love the class. . . . There are none of the power struggles and behavior problems."

Puzzled About Evaluation

She is puzzled, however, about how to interpret what she observes and how to evaluate learning. When she watches group work she sees students talking about math and coming to agreement, but some other observers describe the same scene as students copying from one another. How, she wonders, does one get an accurate and objective assessment of what is going on? Her students' poor literacy skills prevent her from testing them on the sorts of word problems she believes they could solve. However she is pleased by some of the insight they display when they come up to the overhead projector to explain their solutions visually. At present she uses group assessment: students work on the problems together in small groups and then collectively evaluate the contribution each member has made to the task.

Five Colleagues Collaborate

Unlike most teachers trying to innovate, however, Sandy has five thoughtful colleagues collaborating with her to understand her teaching. At Holt High School, the PDS math team includes: Mike Lehman, who teaches full time at Holt and developed a probability and statistics course that Sandy is teaching this year; Bill York, chairman of the Holt High School mathematics department; Janet Wilson, a guidance counsellor who coteaches Algebra 2 with Mike; Perry Lanier and Pam Strahine from Michigan State; and Sandy herself, who teaches in Holt in the morning and at Michigan State in the afternoon. The group is looking closely at the two courses Sandy teaches – Probability and Statistics and Practical Math – and at Lehman and Wilson's Algebra 2. They observe classes regularly, and have interviewed students in practical math in order to learn how these adolescents are understanding the math discussed in class; they meet weekly to talk about what they see and hear.

Bill York reports that his interviews with the students in Basic Math strongly confirm Sandy's impression that the students are "responding positively":

> **Students seem to really like the course. They say they are able to learn more because of the groups, and because of the way Sandy teaches the course. . . . They say they like math, and they seem to enjoy being there. I think absenteeism is lower."**

He says that watching Sandy and interviewing her students has provided him with some real surprises: "I'm a very traditional teacher, so it has been interesting for me to see this. I teach a lot of honors sections, and I expect [students] to be more on task. But it is impressive to see that they learn anyway."

Like Sandy, Bill doesn't feel that he has the evidence to claim that these students are learning more than they would in a more traditional class, but he has found that in interviews in which he has asked them to work in a variety of math problems, "they did much better than you'd expect them to do." The problems required serious thought. One, for example, asked them to find the measure of the angles of a triangle in which the second angle is twice the first, and the third is three times the first. Another problem asked them to think about how fast two painters working together could paint a room if one took two hours and the other took three hours working alone. He was impressed by the strategies they used, and the way they thought aloud about the problems: "They are more free, more willing to share what they know."

Sandy and Mike are also innovating in the courses they teach to college-bound students – using cooperative groups and emphasizing the importance of explaining solutions. Mike team teaches Algebra 2 with guidance counsellor Janet Wilson. Sandy and Mike work together on a probability and statistics course which Mike developed and which is being offered this year for the first time. Although the students in Probability are older than those in Algebra 2, Sandy and Mike mention many of the same challenges and satisfactions when they describe their two courses.

Inflexible Students

The biggest difficulties relate to the somewhat inflexible ideas about teaching, learning and mathematics which successful students have developed over 10 or more years of schooling. For example, Mike, like many of his colleagues in other departments in the High School, reports that many teenagers resist working in groups, arguing that teachers *ought* to lecture to them and tell them how to solve the problems in the book. Recently, says Mike, a student put aside her work to tell him angrily, "This is *your* job." She insisted that groups could never be as effective as a teacher's lecture. As she berated him, Mike recalls, her partner shouted excitedly, "Oh, I get this," and began to explain his new idea to a third student.

Classes that make unfamiliar demands displease some students who have always gotten A's by memorizing formulas for the test.

Mike had expected students to feel some initial frustration – which is one of the reasons he had wanted to coteach with a guidance counsellor – but he had hoped that they would work it through more quickly. Sandy gets the same message from the students taking Probability: "They feel I'm not doing my job because I don't validate their answers."

Classes that make unfamiliar demands displease some students who have always gotten A's by memorizing formulas for the test. "I am trying to *force* them into understanding," says Mike emphatically. "They don't *want* to understand. This isn't [in their minds] the way you learn math." But others thrive in an environment which values reasoning through a problem and explaining your thinking to others. One of Mike's students is taking Algebra 2 for the third time. He is, says Mike, a good thinker, and he has now become the class expert. Although he sometimes has difficulty justifying his solutions in writing on tests, he can always make his ideas clear to his group, and he contributes regularly to full class discussions. "It is," says Mike, "a big change in role for him."

Conversation Important

Conversation plays a leading part in the effort to rethink mathematics teaching in this professional development school. Students talk about mathematics in order to figure out ways to apply what they know to mathematical problems, and teachers open up their classes to one another and talk about what they hope for, and what they see and hear. Some of the biggest challenges relate to the quality of these conversations.

Changing Minds continued from page 2.

designed to ensure standardization. Instead, we have to think of teachers as genuine professionals who need and who model all of the same skills as other workers in the knowledge age economy of the twenty-first century. Children are not less but far more complex and varied than automobiles, computers, or financial systems. Helping them learn is similarly complex. It requires knowledge, judgment, and teamwork. Schools must be organized and managed accordingly. Changes in teachers' work means reciprocal changes in the work of principals and other administrators, making their roles more like those of managers in high-tech organizations and less like those of shop foremen in the old industrial model factory.

To make all of these changes in teaching, learning, and schools as organizations, we need new knowledge – knowledge that is at once theoretically powerful and deeply practical. Generating such knowledge will require far closer collaboration between university-based researchers and public school teachers than the profession has seen over the past half century. Professors need to spend more time in K-12 schools, working alongside teachers as professional colleagues. Reciprocally, teachers need to spend more time engaged in reflection and inquiry. Only through such collaboration and crossover between the university and the schools will we get the knowledge we need to renew education.

The Educational Extension Service was created in the fall of 1988 to support just the kinds of changes in minds and practices sketched above. It consists of two parallel sets of partnerships. First, partnerships between MSU's College of Education and a small number of public schools – these are called professional development schools (see list on back page for specifics) – where teachers and other practitioners collaborate with university faculty to (1) improve teaching and learning for K-12 students, particularly students at risk of academic failure, (2) improve the education of new teachers and other educators, and (3) make supporting changes in both the schools and the College as organizations.

The second set of partnerships is designed to make the results of work in the partnership schools accessible to other schools all across the state. Fortunately, a large number of organizations committed to getting new knowledge into educational practice already exist throughout Michigan. These include intermediate school districts, professional associations, and institutions of higher education. Over time, we plan to form partnerships with many such organizations. By working with and through them rather than setting up an entirely new statewide network, we can reach many schools, teachers, and administrators, and we can do so more efficiently than we ever could if we worked alone. Such a statewide network of networks will take time to build. We have begun working with a few organizations in each category (see list on back page for specifics). Over the new few years, we will gradually expand this set of dissemination partners to cover virtually every corner of the state.

It is both difficult and important, says Mike, to find problems worthy of extended group attention – problems that people *need* to discuss, problems where you can't just find an answer by plugging some numbers into a formula. He cites the example of a problem he posed on a recent test: "Why is $x^a/x^b = x^{a-b}$?" The groups were to try to explain this phenomenon, and failing that, to give some examples they could use in a proof. Some groups managed the task while others didn't, but all discussed their theories with one another. Mike continues to search for problems that provoke and require this sort of discussion. "I've had lots of non-success here – I don't know if this is failure."

While students struggle to explain math problems to one another, teachers search for productive ways to discuss teaching. "It is hard," reflects Sandy, "to develop a culture where we can talk. We tend to go either to the high moral level – 'We all want kids to learn' – or to the real nitty gritty – 'What books do you need by Monday?' PDS is about developing a way that lies in between." ■

Editor: Helen Featherstone

***Changing Minds* is published as a nonprofit service by the Michigan Educational Extension Service which is funded by the Michigan Department of Education. Please address all inquiries to: *Changing Minds*, 116 I Erickson Hall, Michigan State University, East Lansing, MI 48824.**

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- Northwestern Michigan College and the Northern Lower Michigan Leadership, Teaching and Learning Consortium
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Craft Paper 91-3

ASSESSING ASSESSMENT: INVESTIGATING A MATHEMATICS PERFORMANCE ASSESSMENT

Michael Lehman

Published by

The National Center for Research on Teacher Learning
116 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

February 1992

This work is sponsored in part by the National Center for Research on Teacher Learning, College of Education, Michigan State University. The National Center for Research on Teacher Learning is funded primarily by the Office of Educational Research and Improvement, United States Department of Education. The opinions expressed in this paper do not necessarily represent the position, policy, or endorsement of the Office or the Department.

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The National Center for Research on Teacher Learning (NCRTL)[1] was founded at Michigan State University in 1985 with a grant from the Office of Educational Research and Improvement, U.S. Department of Education.

The NCRTL is committed to research that will contribute to the improvement of teacher education and teacher learning. To further its mission, the NCRTL publishes research reports, issue papers, technical series, conference proceedings, and special reports on contemporary issues in teacher education. For more information about the NCRTL or to be placed on its mailing list, please write to the Editor, National Center for Research on Teacher Learning, 116 Erickson Hall, Michigan State University, East Lansing, Michigan 48824-1034.

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[1] Formerly known as the National Center for Research on Teacher Education (1985-1990), the Center was renamed in 1991.

Abstract

The author of this paper describes how he tried several different strategies to enhance his students' understanding of the mathematics he was teaching. He asked his students to work in cooperative learning groups, to give presentations of problems they had solved, and to write about solutions they had found. With all these changes in teaching strategies he began to question if traditional methods of assessing his students were sufficient for testing the type of understanding he was striving for in his students. In this paper the author discusses how in answer to this question he developed a performance assessment as a final exam for his high school Algebra II class. The author explains how he organized the exam, how the students prepared for the exam, and then the results of the exam. He explains how some students surprised him as they performed very well and were able to explain in detail the concepts they were dealing with. Given a chance or when questions were rephrased, students were able to do very well explaining a problem in contrast to drawing a blank on a traditional test and being doomed to a poor grade. Others did not do as well as expected. Even though they had done well in class and had a good understanding of the mathematics he had taught, they were unable to explain the concepts. Instead they relied on memorized facts and simple computation. The author includes some comments judges made about the students and some of the reactions of the students to this type of final exam. The results of this assessments have direct consequences for the author's teaching. The author concludes by discussing the consequences of this assessment, how he will use what he learned from this assessment to improve his students' understanding, and questions that were either unanswered or created by this assessment.

ASSESSING ASSESSMENT: INVESTIGATING A MATHEMATICS PERFORMANCE ASSESSMENT

Michael Lehman[1]

I have taught high school mathematics for 14 years. Over the past several years, I have become concerned with my students' understanding of algebra concepts and skills. Numerous research findings point to the lack of mathematical problem-solving skills and conceptual understandings on the part of our nation's adolescents (National Research Council, 1989, and National Council of Teachers of Mathematics (NCTM), 1989). After my lectures and whole-class discussions of the concepts, my students could perform the computations quite well, but seemed to have very little understanding of what they did and why. When I asked them to explain their reasoning, many seemed unable to go beyond simply telling what they had done to get the answer.

To alter this situation, I have tried several different strategies to enhance students' understanding. For example: To help students discuss and share ideas, I used small-group cooperative learning, oral group presentations of topics and problems, and written assignments about mathematics. My assignments seemed to help students by requiring them to reason about mathematics and justify claims and responses. Discussions in their small groups increased in length, frequency, and quality. I heard students say to each other, "That's fine, but why?" or "We need to put this into words the rest of the class will be able to understand."

With all these changes in strategies, I began to wonder about whether my traditional tests were sufficient for assessing student understanding. In trying to think about assessing student understanding differently I began to focus my attention on what the *Professional Standards for Teaching Mathematics* (NCTM, 1991) says about assessment. An important issue raised by the *Standards* is to "align assessment methods with what is taught and how it is taught" (p. 110). Faced with the problem of assessing what I speculated to be a new kind of mathematical understanding for my students, I began altering my tests both in content and form to find out what my students did and did not understand. I wanted less computation required on the tests and more written explanations about solutions to problems. I have tried group assessments in the form of presentations of problems or topics to the class as a whole. I assessed individuals based on their contribution to the group presentation. I also tried giving the groups a problem to solve and having them write about their solution.

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[1] The author teaches mathematics at Holt High School. He is very grateful to Michelle Parker and Pam Geist for their hours of editing and encouragement.

All of these changes provided me more information (about my students as a group and as individuals) than simple computational tests. However, the changes left me feeling that I was not getting an accurate assessment of what my students *understood*. I kept wondering if my students understood more than I was able to give them credit for, or perhaps less. Furthermore, how could I "see" any lack of understanding in a way that would help both the student and me identify it and talk about it? My concerns were echoed in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), which states, "It is not enough for students to write the answer to an exercise or even to 'show all their steps.' It is equally important that students be able to describe how they reached an answer" (p. 140).

My concern for trying to understand what my students are learning in Algebra 2 led me to design their final exam for the 1990-91 school year as a "performance assessment." Performance assessment can mean a variety of things. The California Mathematics Council (1989) in *Assessment Alternatives in Mathematics* provides alternative ways to think about how to assess student performance. The idea of performance assessment provided an alternative to a traditional exam in which students solve a variety of problems, individually, using paper and pencil within an allotted exam time. The performance assessment offered students the opportunity to discuss a limited number of problems representing a wide range of concepts and to solve them in cooperative learning groups prior to the exam. One of the primary goals in this assessment was to have students be able to justify how they solved the problems. During the first half of the exam period, four panels of adult "judges" each listened to a cooperative group discuss their problem solutions.

While half the groups were doing their performance assessment, in my classroom other cooperative groups wrote about solutions to three problems similar in style to those described above but differing in content. I asked these students to construct their solutions with as much detail as possible, though they did not need to do the actual computations for the problems unless they felt it lent validity to their methods. In the second half of the exam period, the groups changed places.

Background

Holt High School is a small Class-A school accommodating grades 10 to 12. Located in a blue collar and middle class suburb of Lansing, Michigan, our student population is predominantly white. A 1984 survey of high school graduates shows that about 78 percent of the school's students take some college courses. Students are required to take two mathematics courses in grades 9 to 12 in our district, approximately 60 percent take Algebra 2.

Algebra 2 is a yearlong course in which students attend 60 minutes of class per day. In my Algebra 2 course last year, I had 28 students with a wide range of mathematical abilities. I assigned students heterogeneously to small groups in the beginning of the year based on their Algebra 1 and Geometry scores and their reading scores on a nationally normed achievement test.

After I assigned students to groups, I made changes only when I saw that a wider range of knowledge and expertise would help the students better understand the mathematics. Students were not allowed to change groups for social reasons. If a group had a social problem, a counselor,[2] who worked regularly with the class, helped resolve conflicts.

Preparing for the Exam

Three days prior to the exam, I gave my students six problems which drew on the main topics discussed over the year in class. I also gave them an explanation of how the final would be conducted, which read as follows:

> You should find six problems included in this packet. You should work on these problems as a group as well as on your own time. During the exam period you will be asked to explain your results before a panel of judges. Each member of the group will be able to explain each problem by themselves. Other members will be present but will not be able to offer ideas on an individual's problem. You will not know which problem you will be asked so be sure to study all the problems. In your explanations include samples of graphs you may have used, calculations you may have done, charts you made up and any other information you feel will help the judges understand what you know. Do not write a script that you intend to read as this would only prove you can read.

Each problem included some computation along with opportunities to explain and make judgments based on the results of the computations. In these problems the students were either given a situation in which they had to derive data for the problems or they were given a set of data in which they had to decide how to analyze and expand the given information. Creating problems that would provide opportunities to show mathematical reasoning in multiple ways, while also being interesting and worthwhile, proved to be a major task. I worked to design the problems for several weeks along with asking anyone I could find for ideas. With the help of many of my colleagues I was able to write suitable problems.

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[2] The school counselor, Jan Wilson, and I participate in a classroom research project about high school mathematics students' self-esteem and sense of mathematics efficacy.

I include three problems here illustrative of the range of questions students faced.

1. You and your partner have decided to go looking for a buried treasure described on a scrap of paper found in the basement of an old house. The only clues to the treasure's location is the following:

   "The treasure is buried in a spot that is the same distance from the boulder as it is from the railroad tracks. It is also . . ."

   And the rest of the information is missing. But some other clues you may be wise to consider are:

   1) the distance from the track to the boulder is 11 yards.
   2) consider the tracks as the directrix.
   3) keep all of the units in yards or feet.

   Keep in mind the distance of the treasure from the railroad track is interpreted as being the length of the perpendicular drawn to the tracks from the treasure.

2. Hooke's law states that the force F (weight) required to stretch a spring x units beyond its natural length is directly proportional to x.

   You have a spring hanging from the ceiling in the classroom whose hook is 8 ft. above the floor and you want to stretch it down to 3 ft. above the floor in order to hook it to Mr. Lehman's belt loop. Devise a plan to determine how much weight would be needed to pull the spring down. What would you need to consider? Be as complete in your strategy as possible (What steps would be needed?). How would you know if the spring would lift Mr. Lehman or not? When wouldn't it lift him at all?

   Create a set of data to prove your conjecture.

3. Suppose you are a doctor doing research on cancer cells. You have found a certain type of cancer cells are growing as follows:

Weeks	0	1	2	3
Number of Cells	1	4	16	64

> You experiment with different drugs and EUREKA! XI3V causes the cancer cells to stop all further growth and the cells start disappearing at a rate of 10,000,000,000 per hour with a maximum of 5 doses per day. More than 5 doses per day will destroy the patient's liver and kidneys and the person will die.
>
> If you have a patient with this type of cancer and you have estimated that they have about $2.814749767 \times 10^{14}$ cancer cells, how long have they had the cell growth occurring?
>
> How long would you prescribe your patient take XI3V in order to make sure that all the cancer cells disappear? How many doses will this take? Give your answer in a reasonable unit of time (i.e., days, weeks, months, or years; which ever seems to be the most useful).
>
> If another patient comes in who has had this cancer for a year and is only given an estimated 5 years to live unless you can get the number of cells in her system below 10,000,000 within the 5 years so her body can start to repair the damage the cancer has done, would you put her on this medication and give her hope for a continued life? Be very clear in you explanation and have the appropriate figures to backup your determination.

The students had three class days and one weekend to work in their peer groups on solving the problems and constructing explanations that provided support for the solutions. When I first passed out the problems, I expected that students would waste a lot of time in the beginning and get themselves into a time bind towards the end. Yet I was pleasantly surprised; both the observers[3] and I noticed that students used their time very well during the three days. We also noticed that the conversations went beyond simple computation to talk about why different individuals solved the problems in certain ways and what alternative approaches were possible. The students worked very hard but did not seem to panic or be under the tremendous pressure I was used to seeing in traditional reviews for finals. The class seemed to have an atmosphere of seriousness, but also of confidence. Students seemed to believe they could solve these problems in ways they could discuss.

The Judges

I organized judges into panels of three persons that included one person who knew the mathematics subject matter necessary to solve the problem, one person who was not as strong in mathematical knowledge, and one prospective secondary mathematics teacher studying teacher preparation at Michigan State University. The mathematics person could focus on the computation and the logic of students' explanations and mathematical understanding of concepts. The non-mathematics person could focus on the confidence level of the student. As I explained to my students, those judges should have enough confidence after hearing the student's explanation to entrust the student to solve problems for them. I asked prospective teachers, scheduled to student teach in the fall, not only to help in the assessment of the computational algorithms used to solve the problems but also to judge the mathematical conceptual understanding students had. I also wanted the prospective teachers to learn through experience that, even though a teacher may use many strategies to support and develop student's learning and understanding, students may still not conceptually understand some content. I kept myself off the judging teams since I believed I would bring a set of preconceptions about students' abilities and understandings and thereby constrain what I could really "see" students doing and thinking.

---

[3] The observers included Pam Geist, a doctoral student studying mathematics education at Michigan State University, and Jan Wilson.

Before the actual assessment I gave the judges guidelines and evaluation forms for their task (see Appendix). I developed the criteria used on the evaluation forms from similar evaluations I had used during the school year for individual student and group presentations. I believed students would feel comfortable with these evaluation categories since they had seen them during the year. Furthermore, these six categories defined what I was trying to assess about my students' mathematical knowledge without being too burdensome to the judges. I also wanted a form that would not get in the way of students' and judges' discussions. I gave each student a copy of the evaluation form when I passed out the exam questions so they would know what they were being judged on as they were preparing.

I allowed the judges to choose whether they wanted to use the evaluation form I provided or simply give me written comments with a numerical score within the range of points allowed on the form. Most judges found the form very usable and stuck with it, just adding comments to the bottom. A few decided they could better reflect the students' understanding by using more extensive written comments. Both methods seem to work quite well as far as helping me know what my students understood. I found I had no problem evaluating students with the combination of methods used by the judges.

I created two main categories on the form, "Mathematics" and "Presentation," since I wanted to clarify for the students what they were being judged on. The "Mathematics" section reflected what I believe are the four essential characteristics used in solving mathematical problems. These were also the components we as a class had discussed and focused on throughout the year.

I included the "Presentation" section since I wanted the students to know that just doing the mathematics was not enough. They would have to communicate their understanding of the mathematics to the judges. Although I did not want to overemphasize this component (presentation is not more important than mathematical understanding), I did want it to be a part of the process.

I asked the judges to keep in mind my goal for the final exam, which was to understand what my students understood. Also, I reminded them that this was the first time most students had faced an assessment situation like this and, therefore, the students might be nervous. I cautioned judges about using leading questions that might cause students to explain problems without truly understanding them. Yet, I encouraged judges to use probing questions to help students, when necessary, get started on their responses. I wanted students to be able to say something and feel confident that their preparation for this final benefitted them. If students were able to provide reasonable explanations during their discussions, and not just get hints from judges that would make it easy, I could be somewhat assured that students' explanations reflected their understandings.

The Day of the Exam

I reserved the library for the entire exam period (which was, under our school policy, an hour-and-a-half long). I arranged the furniture into four areas so that four groups could be taking the exam at the same time. Each group had as much privacy as possible during the exam. I circulated around the room in order to handle procedural questions if they arose.

During the actual assessment, students went before the panel of judges as a group, but only one student presented a problem. The judges picked which problem each student presented, therefore requiring every student to be able to discuss all the problems and not just the one or two they felt most comfortable with. After the judges heard a student discuss a problem, they would open the floor to other students in the group who wanted to add anything or refute what they had heard. I set it up this way because I wanted to know what each student understood but at the same time I did not want the students to feel totally alone (without the peers they'd studied with) before the judges.

I combined the information I received from the judges and the problems the students did while in my classroom for a final exam score. I used the average of the three judges' scores for two thirds of the total exam score; and the in-class problems counted for one third. These scores were combined to make up 20 percent of the students' semester grade.

What I Learned About What My Students Learned

The results of this assessment gave me plenty of information to digest about my students, my teaching, and our curriculum. First, I learned that many of my students were still only superficially learning and understanding the mathematics. In their groups during the year I overheard excellent discussions about issues and topics we studied in mathematics, and they were also getting better at writing explanations and justifications. However, when it came to explaining the mathematics to the judges, they were only able to tell the steps they took in solving the problem. They fell short when it came to explaining why they approached a particular problems in a certain way. A common response judges heard was, "That's the way we did it in class."

I am still trying to figure out why what we did in class did not translate into better performances during the final. I suspect that some of the students froze in the testing situation despite all I had done to help them relax. Some of the students seemed to have trouble explaining problems individually. They did not have their partners to provide them with some connecting ideas that would allow them to give a coherent explanation of their understanding. Finally, I wonder if the students needed more practice throughout the school year with performance assessments since the practice might help them to understand better what they need to do to prepare and carry out a good discussion of a mathematical problem.

In addition, I was surprised that some of the students I expected to do well didn't, and some I *didn't* expect to do well did! I think some prompting from judges helped these students begin to respond to the questions. While several students did well on their own, others gave good explanations after the judges asked several questions to help them focus their thinking. I felt especially pleased about this finding. If these students were taking a traditional final examination and got stuck on a problem, they would probably be doomed to be unsuccessful and probably continue not understanding. On a performance assessment I could tap into what they actually understood. I also could sort out their misunderstandings from simple computational mistakes common on traditional final exams. The performance judges could help students, through probing, sort out conceptual ideas from mistakes based on incorrect computations.

Another way this exam differed from traditional finals is that I gained information from three different professionals about each of my students. Each judge helped me piece together a picture of my students' understanding that would not have been possible on a typical final. The judges' comments reflected a wide range of observations about what mathematical understanding my students had. Most judges focused on what sense students could make of the mathematics they were doing. Typical of these kinds of comments was this example: "The student was accurate mathematically in solving but did not manifest very deep understanding of what the problem was about." Said another judge, "Started by stating the sense of the problem—the relationship between pressure and volume."

Often judges commented on how explanations and calculations fit together with the problem the student was solving and his/her understanding of it. Here are three comments about one student:

> He calculated the correct equation for the parabola. The only thing he was unable to do was explain the formula [distance] he used to get his equation for the parabola. Other than that his explanations were very good.
>
> Did not understand derivation of formula for parabola—could not provide explanation for why formula works—however set up problem nicely, clearly understood problem.
>
> [This judge addressed the comments to the student] I hope you continue with your agility and explanation based on the graphical representation of this problem. That's important. Push yourself on why the formula/equation works.

The judges seemed to agree that this student could perform the calculations correctly. Yet they all pointed to the student's weakness in being able to explain how or why the formula worked. He seemed to be unable to make the connections as to why he would use the distance formula, though he knew it was necessary to solve the problem. As a teacher I learned that this student knew how and when to use the formula, but could not say why it worked—which is what I want my students to be able to do. What I learned about this student I might not have learned on a traditional final exam.

Another set of comments provided another picture:

> Explained that she is just doing the problems like the book said. [She didn't know why she "logged" things to solve for *x*]. Did not really explain why she did things very well. However, she was able to interpret her results and seemed to understand what they meant.
>
> I asked [student] what a log is, and she said "some number to a power" and could explain nothing more about the concept. Throughout her performance she also kept saying she didn't know if she was "right". All of these comments point to an emphasis on procedures—which for the most part were passable until #3 where she divided instead of multiplied [even after a judge gave her strong prompts].

> [Student] has an attitude problem. She thinks that she really understands more than the other people in her group and she may be partially right but she has a long way to go. When questioned she seems to think that it doesn't matter if she's wrong if it is her opinion. She doesn't seem to realize that in math everything isn't wide open, that there are more than opinions. She worked on problem four and immediately identified logs. She said she doesn't really know what a log is or what it means to log both sides. Most people don't know. She did most of the problem well and was articulate. I couldn't judge her accuracy not having done the problem. One serious error was finding that the treatment would take 1,172 days and dividing by 5 to find the number of doses. When questioned she didn't revise and simply said she might be wrong. A judge asked about 10 days and how many doses that would be. She seemed to understand that 2 was unreasonable but didn't want to think about it at this point. She did do a good job explaining why it makes sense that it would take longer to get rid of the disease than it would take for disease to grow.

These comments give me a lot of information about this student. In class she always offered suggestions and usually could derive a correct answer. Her classroom participation led me to think she understood the concepts very well. However, the judges' comments allow me to see that this student is very capable of doing the computation without having the depth of understanding I had hoped for. On a traditional test she would have made the one mistake with the division for which I would have taken off a few points thinking she had just made a simple mistake. I would never have suspected the depth of her misconception would go to the point where, when confronted with it, she would choose to stay with it even though she would admit it was unreasonable. If I had this information earlier in the year, I would have been able to address some of these misconceptions to work towards further understanding.

From this student's comments I learned that she was able to perform the mathematics and understand most of the concepts in the problem. However, with some of the information she chose to use, she did not understand where it came from. In a traditional paper and pencil exam, with a few lines of computation to illustrate what a student knows, I might have never known that the student did not really understand the distance formula.

While learning about my students' substantive mathematical understanding, I also learned about their affective mathematical views. I learned that students seemed to enjoy this type of assessment. They felt confident they could do it! Afterwards, several students told me that they felt good about the exam and enjoyed taking the final this way instead of working problems for one-and-a-half hours. They felt they demonstrated what they really knew. These responses gave me information usable in answering another question I have wondered about: How can we help our students feel good about themselves in relationship to mathematics? During the past two years I have been working with Jan Wilson in trying to find ways to help our students improve their self-efficacy in mathematics. As stated in *Everybody Counts*, "In the long run, it is not the memorizations of mathematical skills that is particularly important—without constant use, skills fade rapidly—but the confidence that one knows how to find and use mathematical tools whenever they become necessary" (National Research Council, 1989, p. 60). When we talk about student self-efficacy, it is this level of confidence that we are referring to.

Since only some students volunteered their comments, I cannot generalize about all students in the class; perhaps there were several students who did not like the exam but who chose not to tell me. However, I have been teaching long enough to know that if students truly dislike something, they usually let you know one way or another. Also, by looking at the expressions on students' faces during and especially after the final was done, I was able to get a sense of how they felt about it. I did not see the strained and dejected looks I usually see during and after finals. Rather, I saw students who felt they had accomplished something. They were congratulating each other with "high fives" and commenting on how they felt they did. They also offered alternative ways of explaining a problem to their peers that the presenter had not used before the judges.

Several students who normally did not do well on exams were pleased with their presentations. One student commented that he was grateful for the judges taking the time to ask questions since he knew the information but had trouble finding the right words to describe it. This was similar to what the judges said about him. For this student to walk away feeling good about himself in relationship to a mathematics assessment was worth all my efforts to plan and organize it. The next day when he found out he got a "B" on the exam he literally jumped two feet off the ground and went down the hall screaming to his friends.

What Now?

As I enter the 91-92 school year, my task is to use what I learned from this type of assessment as I plan for instruction that will continue to improve my students' conceptual understanding of the mathematics. I now know that good conversations in class do not always transfer into good understanding down the road. I must look for ways to help students transform in-class conversations into meaningful understanding. Transforming what I learned into realistic changes in my classroom is the hard job that lies ahead.

I also have to design a method of doing performance assessment throughout the year. I cannot wait until the end of the year to gather this information since I can better help students' understanding through ongoing assessments. Having four or five performance exams during the school year could help me gain an understanding of my students and allow me to help them be reflective of their growth and change. This would provide me with better checks on their understanding, and how their sense making does and does not fit with my instruction.

I am challenged by some hard questions about my instruction, the curriculum, and general conditions of learning high school mathematics. First, how does a teacher come up with problems that lend themselves to a performance type of assessment? Since the problems require students to think about something, the problems should reflect something worthwhile to wonder about and something real. How does a classroom teacher create problems that fit these requirements around each issue and theme discussed in the curriculum. This set of questions surrounds designing problems that invite discussion.

Another set of questions concerns arranging a performance assessment within the traditional school structures. During a normal school day under normal conditions, I have to find a way to put together panels of judges I will need several times during the year. Where can I locate people? How can I begin to involve the community outside school? Also, without the benefit of the university personnel who work in our building,[4] how does a teacher put together these panels?

I feel very strongly about providing opportunities for performance assessments. The kind of information I received about each student and the reactions of the students make it clear to me that this is a much better method of assessing understanding than typical paper and pencil tests. If I can assess my students' understanding in a more realistic situation and at the same time increase their confidence in themselves in relationship to mathematics, how can I simply rely on only traditional tests?

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[4] The College of Education at Michigan State University has been in a partnership with our school since January 1989. This collaboration is aimed at enhancing the education of practicing professionals (at both institutions), prospective teachers, and high school students. Some of these individuals served as judges are part of our partnership work.

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Appendix

Algebra 2
Discussion Final Guidelines

Please keep in mind that this is a new experience for the students as well as for us. Give the students plenty of opportunity to explain themselves but if it is obvious that they are trying to fake it or are unsure of themselves let them know that it is not what we are after and move on.

Only one student per problem. They have been instructed that they will have to discuss the problem on their own without help from other members of the group. After you feel this student is finished if you want to ask another student some questions about this problem that is fine.

If you pick a problem that a student seems unprepared for, let them do what they can and then come back to that student with a different problem. Please make note of this on the evaluation form.

Please use the following evaluation sheet in assessing the student's discussions. If you find the categories I have outlined unusable or too constraining, feel free to write comments in the comment section or on the back. In assigning the final points you need to be as specific in your comments as possible. Also remember that I will need these forms to discuss their evaluations for any student who wants to check on their performance. If possible, please inform the students of their score. If you can't due to time restrictions and opportunity to confer with the rest of the team, I will be available for them to check grades before school and after on Wednesday and Thursday.

Algebra 2
Discussion Final

Name ____________________

Mathematics:

1) Making sense of problem (Understanding concepts)	1 2 3 4 5
2) Problem-solving strategies (Methods used)	1 2 3 4 5
3) Accuracy of results	1 2 3 4 5
4) Interpreting results (What do the results mean?)	1 2 3 4 5

Presentation:

1) Ability to communicate results (Clarity, use of charts/graphs)	1 2 3 4 5
2) Explanation (Able to answer questions)	1 2 3 4 5

Overall Score

____

Comments:

Craft Paper 93-2

"COULD YOU SAY MORE ABOUT THAT?" A CONVERSATION ABOUT THE DEVELOPMENT OF A GROUP'S INVESTIGATION OF MATHEMATICS TEACHING

Helen Featherstone, Lauren Pfeiffer, Stephen P. Smith, Kathy Beasley, Debi Corbin, Jan Derksen, Lisa Pasek, Carole Shank, and Marian Shears

Published by

National Center for Research on Teacher Learning
116 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

September 1993

This work is sponsored in part by the National Center for Research on Teacher Learning, College of Education, Michigan State University. The National Center for Research on Teacher Learning is funded primarily by the Office of Educational Research and Improvement, United States Department of Education. The opinions expressed in this paper do not necessarily represent the position, policy, or endorsement of the Office or the Department.

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[1] Formerly known as the National Center for Research on Teacher Education (1985-1990), the Center was renamed in 1991.

Abstract

In this report, the authors describe the first year of an ongoing intervention study and the learning of a group of teachers and researchers who are working together to make changes in the ways they teach mathematics. When reflecting on their beginning efforts, the researchers found that several assumptions in their planned interventions were not born out by the group's experience. The subject matter content (integers) they selected as an initial focus of study posed major difficulties for several of the teachers. The teachers' interest in talking about their own practices strongly influenced the group's interactions. Collectively, the teachers and researchers created a learning community that was grounded in watching videotapes of mathematics teaching in a third grade classroom and discussing ideas about teaching and learning in ways that were different from their own experiences as teachers and students. Here, the researchers pose a set of conjectures that serve as a framework for the continuing collaboration of this group of educators' inquiry into non-traditional approaches in the teaching and learning of mathematics.

"COULD YOU SAY MORE ABOUT THAT?"
A CONVERSATION ABOUT THE DEVELOPMENT OF A GROUP'S INVESTIGATION OF MATHEMATICS TEACHING

Helen Featherstone, Lauren Pfeiffer, Stephen P. Smith, Kathy Beasley, Debi Corbin,
Jan Derksen, Lisa Pasek, Carole Shank, Marian Shears

Helen Featherstone, associate professor of teacher education at Michigan State University, is a senior researcher with the National Center for Research on Teacher Learning.

Lauren Pfeiffer, a doctoral candidate in educational psychology at Michigan State University, is a research assistant with the National Center for Research on Teacher Learning.

Stephen P. Smith, a doctoral candidate in teacher education at Michigan State University, is a research assistant with the National Center for Research on Teacher Learning.

Kathy Beasley is a third grade teacher at Averill Elementary School in Lansing, Michigan.

Debi Corbin is a third grade co-teacher at Averill Elementary School in Lansing, Michigan.

Jan Derksen is a third grade teacher at Bath Elementary School in Bath, Michigan.

Lisa Pasek is a sixth grade teacher at Mayfield Elementary School in Lapeer, Michigan.

Carole Shank is a second grade teacher at Averill Elementary School in Lansing, Michigan.

Marian Shears is a middle school teacher at Kinawa Middle School in Okemos, Michigan.

I remember walking timidly into the first session of the Investigating Mathematics Teaching class. I had been intrigued by the description of the class which would be looking at teaching math in a third grade classroom. I had just started my first teaching job, a part time, temporary position in a Professional Development School. I would be teaching math and since I had graduated more than 10 years ago and had only a bad experience in a masters level math course, I decided I better find out "how to teach" math. I knew I wouldn't be satisfied to only use the teacher's guide. Little did I know that a year and a half later I would still be meeting with this group of educators.

—Jan

During my undergraduate work at MSU, I had profound "rebirth" in the area of mathematics teaching. After two years of teaching, I returned to MSU to begin graduate studies and underwent another refocusing—a kind of "a-ha" experience. I realized that I was missing something: the element of genuine discourse and deliberation about mathematics among students. In spite of this experience which was, at the time, quite shaking, I came to the IMT group the following fall feeling refocused and confident that I now had a handle on what I wanted for my mathematics students and for myself.

I was intrigued with the idea of utilizing multimedia capabilities organized around a mathematics classroom. I envisioned independently manipulating the HyperCard facilities to find out "what a teacher could learn" from using such technology.

I had no idea that the technology would serve first as a springboard and later only as a backdrop to the very personal and collective professional investigation of mathematics teaching and learning.

—Lisa

Jan and Lisa are remembering the feelings they had the first time they came to Investigating Mathematics Teaching (IMT), which had been described to them as an experimental course for practicing teachers. Helen, Steve, and Lauren, researchers with the National Center for Research on Teacher Learning, had organized this course in order to learn about the ways in which a multimedia collection of materials documenting teaching and learning in Deborah Ball's third grade mathematics class might be useful to teachers who were interested in thinking about new ways to teach math.[1] They advertised a "group independent study" for elementary and middle school teachers in the masters program in Michigan State University's College of Education, but also welcomed teachers who wanted to participate without enrolling for course credit.

As the recollections of Jan and Lisa suggest, the seven teachers who became the IMT group assembled with different agendas, hopes and fears. They were all, however, committed to thinking hard about the teaching and learning of mathematics. And like Steve, Helen, and Lauren, the three university people, they felt sure that they wanted to teach math in ways that were different from those they had experienced as students in elementary and secondary school. They were, however, in different places on the journey away from traditional mathematics teaching and toward something else. And they were in quite different places then, on that afternoon in early October, 1991 than they are today.

Much of the history of the group is the history of individuals rethinking their own practice and their relationship to and attitudes toward mathematics. Some parts of that story we have told elsewhere (see, for example, Featherstone, Beasley, Corbin, Shank, and Smith, 1993; Featherstone, Pfeiffer, and Smith, in press; and Pfeiffer, Featherstone, and Smith, 1993). Some of it, however, is the story of an ongoing and evolving conversation about the teaching of mathematics that has lasted through most of two academic years. In December of 1991, at the last meeting of the "group independent study," several of the teachers expressed an interest in continuing to meet. Steve, Lauren, and Helen were also eager to continue the work that we all seemed to have begun and so we set a schedule of biweekly meetings for the following quarter. Over winter break all the other teachers decided that they too would like to continue the connection. At the end of winter quarter, we decided to continue through spring quarter, and at our last meeting, in June, 1992, we agreed to reassemble in late August to discuss plans for launching the 1992-93 school year. We have been meeting at least every other week this year.

By the end of the 1991-92 school year, several of us had begun to comment on changes in the nature of the conversation that occurred at our Thursday night meetings. Over the summer Lauren, Steve, and Helen decided to examine these changes empirically and to try to understand them better. In January 1993, they asked the teachers to help them to think about the changes that had occurred over time; several intriguing conversations followed. The conjectures that we[2] generated in these conversations included the idea that, over time, the teachers had begun to "push each other more." In an effort to develop a clearer understanding of what this meant and how the ecology of group meetings might support or discourage this "pushing," we began to look at one meeting through the lens of "discourse analysis (Tannen, 1989; Coultard, 1992; Shultz, Florio, and Erickson, 1982; Cazden, 1989)" and to plan further study of the evolution of this phenomenon over time (Pfeiffer, Featherstone, and Smith, 1993).

Discourse analysis provides one tool for looking carefully at aspects of the group's conversation. It helps us to see, in the actual talk, the ways in which the group members participate in a collective study of the teaching and learning of mathematics and, in turn, support one another's social, emotional, and intellectual efforts to make changes. We learned what some aspects of the discourse looked and sounded like, and how the group worked together to understand, for example, the mathematical and pedagogical questions surrounding the efforts of third graders to grapple with the relative size of one-half and one-fifth. We did not explore, however, the way individuals experienced our meetings, or the way they thought about the relationship between what we did on Thursday evenings and what they did in their classrooms. Hoping to capture and communicate some of this, we returned to our usual tools: writing and conversation. Jan, Lisa, and Marian volunteered to write about some of their thoughts—about their own participation in the group, about the relationship between our joint conversations and their teaching, about the evolution of the conversation. The nine of us then came together on a Sunday afternoon to read and to talk about their texts. Helen then edited the texts and the transcribed conversation into this paper.

Lisa and Jan's writing (p. 1) took the group back to the moment when teachers and researchers came together for the first time. During our first meeting we watched a videotape of a third grade mathematics class discussing a few of the number sentences that the students had generated in response to Ball's request that they "write number sentences equal to 10." One student's suggestion that "200 take away 190 equals ten" launches a debate which remains unresolved by the end of the period: When a second student comes to the board and shows why she thinks that 200-190=190, many of her classmates agree although some do not. For the next three math periods the students work on and discuss problems that their teacher creates and poses in order to explore and challenge the conceptions that underlie the approach which leads the third graders to assert that "you can't take nine from zero so you write down the nine." Our conversation begins with Jan and Lisa's written recollections of what they said and felt during these early meetings.

Our conversation did not, however, move from these early memories into a linear history of the group or of our memories of it. Rather, these texts—some of which are reproduced here in italics—launched us into some reflections on schools, teachers, and administrators, and on the reasons that the teachers have found the group useful and even necessary.

The development of this Sunday afternoon conversation mirrors that of the IMT group. Like the group, it starts with some texts: The texts here are the writings of Lisa, Jan, and Marian. The texts for the IMT group were, to begin with, videotapes and other materials documenting teaching and learning in Deborah Ball's third grade mathematics class. In recent months we have sometimes started watching videotapes of our own math classes. Like the conversation in the group's regular Thursday night meetings, this conversation moves back and between these texts and other matters. And like the conversation in the regular IMT meetings, this conversation ends up digging deeply into questions that come up because of the texts but are not always directly related to them.

GETTING TOGETHER

Kathy: When I read what Jan wrote, I just started thinking about the first time I walked in. And "timid" is a good word. I had this great fear that everyone here was going to be really good at math. I knew I wasn't, and I kept thinking, "What are they going to think when they find out that this person is in this math group that doesn't know anything about math?"

Carole: I was really confused about why I was there. I knew I waited to do something about math, and Kathy and other people talked about the NCTM *Standards*. I didn't know a thing about the *Standards*, plus I didn't know much about math.

FIRST IMPRESSIONS: WATCHING TAPE

When I first watched Deborah Ball end her math class "in the middle of a problem" I panicked. How could she do this to these children? Would they be able to sleep at night not having heard the answer? Is one problem a day enough? How would I ever get through all my "material" if I taught like that? Where are the manipulatives? How can you teach negative numbers to third graders? Why would you?

In some of our early session as I recall the others talking about "teaching this way," the "Standards," and "discourse in the classroom." "This way of teaching" was even compared to the "Whole Language" method. This was all new to me and I was afraid to speak up in the beginning. I spent a lot of time listening and thinking. I often heard doubt in some of the other voices. Some of this had to do with a *lack of support in their district but I also sensed that it had to do with the fact that this was a new way of looking at teaching math. Maybe we were all "walking on thin ice."*

—Jan

My first response to Deborah spending 3 days on 200-190=? was to turn to one trusted friend in the group and say, "I can't believe she spent 3 days on those problems!" As the group discussed the lesson I commented that administrators in my district would have a very hard time with this idea.

Late in the school year, I talked of my own perspective on the 200-190 problem as being an important reference point in the changes in my own thinking. Seven months after that restless, urgent interjection (that my administrators wouldn't understand spending 3 days on one problem), I found myself acknowledging that regardless of opposition or resistance, real or perceived, that I had been experiencing in my teaching, it was ME who had had so much apprehension and conflict with this kind of genuine discourse as a primary pedagogical tool.

Now, I can't believe how much I've changed. I've come to see what really happens in a classroom where students use discourse to construct their own understanding. To a large extent, I had used my district administration as a scapegoat for why I couldn't "do" that kind of discussion in my mathematics classroom. By this later point, a trust has developed in the group that has enabled me to put out on the table that I had been unwilling to say "I have a problem with this. . . ."

But it wasn't just a matter of emotional support, but also of providing an environment for intellectual exploration.

—Lisa

As a beginning teacher I was in essence starting with a clean slate. I was able to make my own curriculum decisions and I had no one watching over me. This was actually a frightening experience. It seemed that the other members in the group had either been teaching for several years and "needed" to make changes in their teaching for survival or they had "grown up" with this way of looking at teaching. I wasn't sure what I thought or believed. I had no foundational experiences to compare this to. I didn't have a store of negative experiences nor of positive teaching experiences. A lot of teachers at my school were looking at math in different ways but I can still remember being advised to "just use the teachers guide your first year."

I remember those early experiments with "testing the ice." I bought math journals and, borrowing from Deborah Ball, I asked my third graders to "write number sentences equal to 10." I was amazed by their responses and by what I learned about their understandings. I looked forward to sharing this with the rest of the group.

—Jan

Jan: I agree so much with what Lisa wrote. Because I think she is saying that so much is self-imposed, and I feel that way, too. I actually get support from my principal,but not from my colleagues. When I say, "Have you tried this or that?" it's as if I don't exist because I don't have credibility. I think I had some feelings reaffirmed here. And so I began thinking that maybe I *am* making some right decisions, decisions that are best for kids, regardless of the "What chapter are you on?" questions. and I've decided, even though it's frustrating and I hope that maybe someday we'll be able to engage, that right now I have to sort of do my thing, and talk to the one or two people who want to talk.

Kathy: Do you think it's partly because you are a first year teacher?

Jan: Probably. They've been around. . . .

Marian: Lisa used to talk a lot about the feeling that no one would listen. No one.

Lisa: I think that they listen, but. . . . A teacher in my district is really struggling to make changes, and she made the comment, "For twenty-five years I've tried to do everything that they've asked me to do and it's never been what they've wanted." It was terrible hearing the pain in that statement.

So often teachers will look and they'll say, "Here's a teacher who's enthusiastic, they're dedicated to their own professional development; they're inexperienced, obviously. They don't know what teaching is all about yet." That's why it is so revitalizing to come here and see that there are teachers here who have been in the classroom for twenty-five years, they know the ropes and they're still fighting.

Kathy: It's true. I remember when I first started out, if I'd make a comment that was discouraging, or not optimistic, people would say, "Now you sound like a real teacher!" It was like they were saying, "Now people will listen to you."

Jan: That's what makes this group unique. Teachers, we beat each other up. We aren't supportive of one another, that's my experience. It's as though there's a competition: Who's going to have the cutest bulletin board display in the hall? or whatever.

The IMT group is a place in which we began to share our frustrations and insecurities. We came to accept that teaching is struggling—of an honorable, honest sort. We also continued to examine interactions and issues in the videotapes and connect them to our own teaching, our own experience and ideas. We began to question episodes of mathematics teaching in the videotapes and echoes of those questions reverberated in our own heads, pushing us to ask such questions of ourselves and of each other.

The group has provided me with a safe environment in which I can discuss my own teaching, listen to others, and force myself to think about and question what I am doing. But most important, this nurturing and challenging atmosphere has encouraged me to take risks.

—Lisa

Lisa: I have a time every week where about half of the kids go out of the room to band. The ones who are left seem to be the ones who have more trouble with math, so I talked to them last week about having a math support time. I had them break into groups in which some people would be giving support and others would be receiving support.

I was watching two students who were working together and one was *pushing* the person who was having trouble, actually pushing on them. It was very vivid. And I said, "Stop!" I held up my pen and I told them that there's a difference between pushing a pen and sending it flying, and just supporting it. I asked them to think about the difference. And that developed into thinking with the kids about what you do when you help someone ride a bicycle: You hold that back bar and you run with them, and you don't know what to say that's going to help, you just encourage them, saying, "You can do this." You're giving the support you can, but you can't "teach" them: The other person really has to get it for themselves.

Anyway, at the end of the period I had them spend the last five minutes writing about what they learned, or what they were thinking. And this one boy wrote, "I learned that *getting* support is much harder than *giving* support." And I thought he had unlocked the mystery of the universe

And I sort of thought that the bike analogy kind of connects to what we do here. Even though we are not world class cyclers, we can still help each other, balance each other out. Even though we are not experts.

Kathy: It's harder to be on the bike, trying to learn, than to be running alongside.

Helen: And it makes you think about yourself and what's involved in accepting support.

THE DEVELOPMENT OF DISCOURSE

Our group began as a collection of individuals who were all interested in examining the teaching of mathematics. Some of us knew each other (three even taught at the same school), and some of us met for the first time at our first session. We all thought that we were simply participating in a ten-week university class. But what happened in that class over time changed our perceptions and our goals.

At first our meetings centered around watching and discussing videotapes of Deborah Ball's third grade mathematics class. The instructors selected the tape each week, and we discussed what was happening in that class and how it pertained to our own classrooms. Through these weekly discussions a pattern began to emerge. At first we focussed on what was happening in Ball's classroom: what representations the teacher used, how specific children made sense of problems, et cetera. Over time, however, our focus began to shift. Instead of looking just at what *was happening, we also began to look at* why *that might be happening. We, collectively, took a step back to look at the bigger classroom picture.*

As a part of this new way of looking at the tapes, we began to examine the patterns of classroom talk: the kinds of questions the teacher asked and the kinds of responses kids gave to her and to each other. Specifically, our group started to focus on the classroom culture and the part that discourse played in it.

By discourse we meant more than just classroom talk. We meant that students were involved in explaining their ideas to each other, that they had, and shared, reasons for their ideas, and they listened to each other. Good discourse requires an environment where students don't look to the teacher for "answers," but look to each other (and within themselves). They don't just accept answers blindly either; they ask for (and offer) logical reasons.

As our group examined discourse in Deborah Ball's classroom, we began to change in subtle ways. We found ourselves asking each other the same questions that Ball asked her students—questions like "Why do you think that?" What do others think?" or "Could you say more about that?" We began to push each other's thinking in ways that Deborah pushed her students' thinking. In other words, while studying the discourse on the tapes, we created a classroom culture of discourse within our group.

This development of discourse seemed to have a dual relationship with trust within our group. First, we were able to develop it because we were beginning to trust each other. But our trust also grew as a result of participating in discourse, perhaps because respect for another's ideas is inherent in good classroom discourse. Interestingly, no one in our group ever talked about the parallels that were developing between our group's talk and the talk in Deborah Ball's third grade classroom. We weren't consciously aware of what was happening, but we were modeling something that grew out of our shared experience.

We did, however, openly discuss how we could cultivate discourse in our own classrooms. We wanted our students to question each other, and to give and expect reasons for their thoughts. As teachers, we tried to model discourse for our students. We also taught it directly, saying things like, "It's important that we listen to each other."

Perhaps we were better able to teach and model good discourse only after we had experienced it ourselves.

It is difficult to trace this experience and it varies for different members of the group. But, there is a shared sense that we moved from a set of concerned individual teachers to a collective that seeks and supports a critical but trusting atmosphere in which we purse an *emerging shared vision of mathematics teaching. This vision includes fostering a classroom culture of discourse much like the one we have experienced.*

—Marian

Lisa: When I looked at this description of the way we developed discourse in here, a couple of things struck me. One was where she says "Good discourse requires an environment where students don't look to the teacher for 'answers' but look to each other (and within themselves). They don't just accept answers blindly either; they ask for (and offer) logical reasons." I crossed out "students" there and put "teachers," because that begins to describe the discourse within our group.

Then, further down here, it says, "Perhaps we were better able to teach and model good discourse only after we had experienced it ourselves." I felt very strongly about that and thought a lot about comments that we've made about how the support that we've felt, the support we continue to feel, supports the development of what we do in our classrooms.

Debi: I was struck by that same thing, because the thing that I realized was that I really didn't know what discourse *was* and I was trying to create it in my classroom. And here we have done it ourselves, slowly. It hadn't really occurred to me that that was what we were doing in here, but we have kind of taught ourselves, or at least I have been taught on my own level, so maybe I can take that and transfer it more easily to what I do with my students.

PUSHING EACH OTHER AND DEVELOPING TRUST

At our first meeting of our second year, I was sharing my own ideas about whether to start the year with a set of lessons which would engage students in a very "safe" introduction to discourse—focusing on setting classroom norms. The students would be encouraged to agree or disagree with each other's puzzle models given very cut and dry criteria. Totally unexpectedly, Kathy asked a question that came at me from across the table like a bullet: "But, isn't the discourse in the task?" THUMP. And I think to myself. . . she's right. If my task is well constructed, won't students engage in a more genuine discourse? But it is the beginning of the year, so maybe we can start off in a safer way, introduce norms and vocabulary in a set of lessons which would require less intellectual risk-taking for these new sixth graders who have never been asked to think about whether they agree or disagree with a mathematical idea.

Already we were challenging each other and ourselves to not take anything for granted, but to really dig into what decisions we were making and why we were making them. This intellectual pursuit of teaching could only take place in a setting of trust. In this context, a hard hitting question is not meant to embarrass or demean—It is a supportive push—Often these questions are ones that are hard to push ourselves on because we may too quickly settle for our own perspective.

—Lisa

Kathy: I don't remember even asking that question.

Lisa: You *don't*?

Helen: [to Kathy] I thought *Lisa* had asked it because the next week you told us that you had gone home after the meeting and said to yourself, "Oh, okay, now I understand: The discourse is in the task."

Kathy: That makes sense to me, Helen.

Helen: But, actually, it turns out that *you* asked the question.

Kathy: I think that's an example of trust. Lisa was telling a story, almost in a way like she's kind of telling us how to do discourse or get ready for discourse. And it just made us all think about it. I don't know. I don't think you ask questions like that to people if you don't think that it's going to be okay with them.

Like, if I ask a question in my staff meeting, people generally assume that I'm asking it not because I'm seeking information, but I'm sort of maybe trying to put their idea down, or play out a problem with it. And I think here, when we ask questions people understand that what we want is information.

Marian: You just said it: "Assume." We don't assume the motive. If something doesn't strike us right, we say, "What do you mean by that?" or "Are you saying that . . . ?" So instead of assuming negative motives, we try to clarify.

Jan: Do any of the rest of you find yourselves, when you're talking with people, saying, "I'm not trying to be argumentative?" I find myself saying that a lot, because I do ask questions. I want to understand people's reasons.

GETTING TO TRUST

Debi: I want to know how we got to this trust. Because it's really hard for me to get to a point where I feel free to talk. I was talking to another instructor about why I don't talk in that class, versus why I do talk in here. She was wondering why, and I'm not sure why except that we've been together longer and I was allowed to be quiet for as long as I needed to be, until I felt safe enough to start sharing things.

Marian: I think too, that the shared experience of watching and discussing the tapes of Deborah Ball's class was a big part of it. We were building a common frame of reference: We could always say, "Like in Deborah's class. . . ."

Lisa: And maybe that common frame of reference was a safely net: We were talking about things that we were really thinking through in our own classrooms when we were talking about what we saw in the other class. We could say, "Look at how directive she was," without saying, "You're being directive," "I'm being directive." We could say, "This person on this tape that isn't here is being directive," and we could discuss whether or not that's okay without turning it into something personal.

Marian: So part of that trust is because we could take those risks with someone else first. Deborah took those risks for us.

WHAT ARE QUESTIONS FOR?

Lauren: I wanted to pick up on Kathy's comment that when questions were asked in the group it was to find out more information. I wanted everybody to talk more about that, because that surprised me: I would have thought questions also served other functions besides getting information.

Helen: I think what she was saying is that they're not a backhanded way of criticizing. That we assume a wholesome motive, if you will.

Debi: And that if you want to know something, it's safe to ask.

Marian: What I think you're saying is that we also use questions in another way: Maybe we ask questions to get each other to think about things from a different perspective.

Lauren: I guess that's what I'm curious about. Is that true? And how do you think about that?

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Marian: I was going to say that I think they're related. That we're pushing each other, but it's with the assumption that it's really in there. We're not saying, "This is what you don't know," or "I don't think you've thought about this." We're saying, "Have you thought about all these angles?" or maybe we're pushing it a little, because we realize from our experience that pushing does help us clarify for ourselves. And so we're helping each other to clarify.

QUESTIONS AND THE CO-CONSTRUCTION OF AN IDEA

Lisa: And we are helping to develop an idea in progress. I think it's really fascinating that Kathy didn't remember asking the question ["Isn't the discourse in the task?"] that drove my journal entries for weeks. I had pages and pages that I wrote about this question; it really pushed me to think about my rationale.

You asked that question and later on you were saying, "I understand now," as if it were somebody else's idea. It was kind of this in-progress thinking that you threw out, "the discourse in the task." And then Carole built on it and it became this idea right there on the table and we really looked at it and it still is really with us, pushing us. And then it kind of came back and you left with your own new version of what the question was. It was kind of like what Helen talked about when she said that our ideas are not just celebrated, but people grab onto a half a conjecture and run with it. Especially in this instance, where you threw out something that just seemed so profound.

Kathy: I think questioning is several different things. I want to revise. Jan made me think about the questions I ask in my classroom, and in this group as well. Sometimes I ask students questions because I want to understand what they're thinking. I don't want them to change what they're thinking. I don't want them to move beyond where they are, I'm just trying to understand it. Sometimes I ask them question in order to make them think harder about this or to move them with their reasoning. Probably there are other reasons.

Marian: And that would hold here in the group too?

Kathy: I think so.

I think about the conversation we had the night when Steve was taking his very strong stand on the multiplication tables. I'm thinking about the questions I was asking him that night: Some of the time I wanted to know what he was thinking, and some of the time, my questioning was to say to him, "Stop and think about this, Steve."

Lauren: So that's not *just* understanding; that's trying to push him to think harder?

Kathy: Yeah. It almost feels like there's a third one, like . . .

Lisa: One is "Keep going with that idea, say more," and other is "Stop and look back on it."

Lauren: Another word that as come up in the writing and the conversation is "challenging," and I've been trying to think if pushing and challenging are the same thing.

Kathy: I think they're different.

Lauren: How would you define the difference?

Carole: Kathy, in your class when you say, "I want you to look at a new idea that's on the board. I want you to think about it," in a sense aren't you challenging them?

Kathy: That's more like pushing.

Kathy: Challenging is almost . . .

Helen: Confrontational?

Steve: It's a way of disagreeing?

Kathy: But it's a *strong* way of disagreeing. I think you have to have a sense that people are going to stay with you and not take offense and get angry before you go to challenge them.

Jan: Are you sort of thinking of challenging and pushing an idea?

Kathy: I think it's gentler.

Lisa: There are supportive questions, and then there is pushing with support. But we have had some discussions where there is some challenging going on, and, for me, it was uncomfortable. That assessment discussion that we had was very uncomfortable for me. I think I probably said two words that whole night. I didn't want to be confrontational, but I was frustrated about how to ask questions without being confrontational. There was a lot of confrontation going on and it felt like it was backhanded so I kind of retreated.

Lauren: So that wasn't a time that you had a sense that the interaction was supportive pushing?

Lisa: I didn't feel that it was supportive, so I backed out of it. But other people were still engaging it so they may have thought it was supportive.

Kathy: I thought it was a great discussion and I just loved it. But now I'm wondering, "What did I say? How did I say it?"

Lisa: It could have just been where I was at that particular day: I was feeling urgency about figuring out *today* what I was going to do for *this* marking period, and it was a very theoretical discussion. That was probably frustrating for me, wanting to have answers or some feedback about what might I do this week when I'm calculating grades. Really, you know, it was more about questioning answers and not about answering questions. But if we say the group is about questioning answers and not answering questions, that's what it was.

Kathy: I guess I left feeling that it was unresolved. I didn't think people had sanctioned anything.

Marian: I never felt like we had to come to an answer. So I guess what I'm saying is that that's why I didn't view it as a confrontational meeting. I wasn't looking for an answer. Maybe Lisa was looking for an answer.

Steve: Maybe what made it seem confrontational is that it didn't seem like there was as much open thinking going on as usual. On other occasions when people are pushing each other, there's more listening and wondering. I didn't seem like there was any change taking place.

Lisa: It felt like we were talking at each other and not with each other.

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Jan: I was listening more than talking, because I'm trying to resolve a lot of these things in my own mind. When I grade, I sort of hide and don't let anyone else know how I do it. I got my school to go away from A, B, C, D. So I just liked to hear the rationale. I have a parent who says, at every report card, "I don't agree with this form of grading, I want to know how she has improved." So I wrote up an explanation that I thought would show him that, but he wants to know if she has gone to a 98 from a 95!

IS CHALLENGING BAD?

Jan: Are we assuming that we all think that challenging is negative? Because it seems like we are using the word that way. I wonder if some of the negative feelings that we might have about this are because, often when you are challenging students in your classroom you're trying to control their thinking. I mean, you're hoping they'll go in a certain direction.

Helen: You don't want them to conclude that division is commutative.

Jan: Well, if you're challenging their thinking about multiplication, your goal is to try to get them to come to some understanding about what multiplication is. There is some control because *you're* asking the questions, and they're not just questions that are out there somewhere and are meaningless. So, I wonder whether challenge implies control.

Lisa: With wrestlers, you have a prize winner and you have a challenger who is hoping to take control over the prize. The word "challenge" has the connotation that there is going to be a winner.

Carole: I drew a mountain: I think of a challenge as going beyond, it's not getting at just the basics, it's going beyond.

Kathy: I guess I want to know what people think: Do you think confrontation is bad, then, not a good thing for our group to do? I feel that reaching a point where we can actually confront each other and be challenging is a *good* place to be. It implies trust.

Jan: It's our ideas that are being challenged and not our *being*. But we're so used, as teachers, to feeling that when our ideas are being challenged our very being is being torn apart.

Kathy: We can't separate ourselves from our practice.

Jan: But maybe we can do that here, right?

Kathy: But I think conflict deepens a relationship. It might be kind of uncomfortable when you're in it. . . .

Debi: Then there are attack questions, and those you don't want. I don't think anybody feels safe in that sort of conflict. But confrontation, to me, is okay.

Kathy: See, I come from a family where people are pretty confrontational and do a lot of attacking, but we've always kind of enjoyed that. But a lot of people don't, and I'm always kind of surprised by that—well, I'm not surprised anymore, but I forget.

If anything, I feel bad that I didn't notice that Lisa bailed out.

Lisa: Debi is saying that she was allowed to stay quiet until she was ready. I was allowed to stay quiet because I didn't feel comfortable engaging at the level that other people were engaging. But I was still engaged. I was still very much compelled by the discussion, but I had just retreated to a safe place where I could get a handle on what was going on because I didn't feel safe.

But it's one thing to be putting something on the table and saying, "Do you think that this is true?" and talking about that idea. And it's another thing to have "This is what I think," "This is what I think," and having a *butting* of ideas instead of a *meshing* of ideas. That was what was frustrating: I was hoping that there would be a collaboration, a wondering about what are some ways to handle this. I was not looking for an answer like, "That's what Teri does, so that's what I'm going to do." I was looking for intellectual exploration.

Steve: It occurred to me that when Kathy was talking about all the ways *we* question and about the teachers in the staff room misinterpreting questions, I wonder how *kids* interpret questions.

Debi: Because kids say to me, "Why do you always ask me why?"

I'm wondering if we think about what assumptions they're making about what we're saying. I kind of wonder how they're feeling. They need that silent time. I put them on the spot.

CIRCLING BACK

Kathy: I want to go back to what Marian wrote about the development of our discourse and say quickly about something I disagreed with: I don't think we developed discourse because we saw it in action on Deborah's tape. I think we developed discourse because of our task. I think the discourse is in the task, and I think that's why our group has discourse, not because it's something we learned from watching Deborah's tape.

Marian: It can't be both?

Kathy: It could be. But that part isn't in here. It makes it seem like we were pretty passive, that somebody taught us how to do discourse, and then they did discourse. I think we were much more active in that. Because of our task, which was understanding our own teaching, understanding our own mathematics.

Marian: I would agree, I would say it's both.

Kathy: Because I'm thinking about my own classroom. They don't watch how to do discourse and then learn how to do discourse. If I give them a good task, they do it.

Marian: The discourse is in the task?

Kathy: Now it's not a question, it's a declarative sentence. So that's what I'm disagreeing with.

Helen: Well, do we need a group simply because we're trying to do something that's hard and different, or is it also because we are trying to create conversation, and we need to engage in conversations in order to create them?

Kathy: Lucy Caulkens says that teachers can't really understand how to teach writing unless they write themselves. It doesn't quite fit for me here, but I guess it must be true.

Helen: I wouldn't agree that it *must* be true. Maybe it's too pat.

Kathy: I mean, it seems sensible: You can't teach writing well unless you write, you can't have good discourse unless you . . .

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Lauren: But I think that the function of the group is broader than that. We are learning more than just about how to create conversations in classrooms, and maybe that's why it's too pat to make it that simple. It makes sense. But it sort of discounts the power that we're learning about, the power that the group provides. But then when you come back to say, well, could you learn to create conversations in the classroom without a group, then that's clear.

My other question was, is the need for a group specific to mathematics? Is it related to the fact that we all had bad experiences learning math?

Kathy: I don't know, I mean, I want to change how I'm teaching reading, and I've been thinking about it a lot this week, but I thought, "This is going to be really hard to do all alone."

I think it's all the same thing.

Notes

[1] For a detailed description of the agenda of the researchers and the early history of the group, see Featherstone, Pfeiffer, and Smith, in press.

[2] The nine authors listed here, plus one other group member who has been unable to participate in the writing of this paper.

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College of Education • 116 Erickson Hall • Michigan State University
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10/4/93

CHANGING PRACTICE: TEACHING MATHEMATICS FOR UNDERSTANDING

A PROFESSIONAL DEVELOPMENT GUIDE

Developed by

Steven A. Kirsner, John S. Zeuli,
Linda Alford, and Michael J. Michell

The Teaching Mathematics for Understanding
Video Project
National Center for Research on Teacher Learning
Michigan State University
East Lansing, Michigan

INTRODUCTION TO THE VIDEO

A. Students' Role as Learners of Mathematics

B. Mathematics Content

1. Mathematics as Problem-Solving
2. Mathematics as Reasoning
3. Mathematics as Communication
4. Mathematical Connections

C. The Teacher's Role

1. Worthwhile tasks
2. Classroom discourse
3. Learning environment
4. Systematic analysis

ORIENTING QUESTIONS FOR VIEWING THE VIDEO

Questions about the Students' Role

1. When you observe the students in the video,

   a. What are students saying that you find particularly interesting or considerably different from what students typically say in conventional mathematics classrooms?

   b. What are students doing that you think is different from what one typically sees during mathematics instruction?

ORIENTING QUESTIONS FOR VIEWING THE VIDEO

Questions about Mathematical Content

1. The NCTM *Standards* suggest that students should engage in worthwhile activities. How would you define the content these students are thinking about and how does this content differ from the content offered in your textbooks?

Questions about Mathematical Content
(cont.)

2. The NCTM *Curriculum and Evaluation Standards* recommends that mathematics curricula at all grade levels focus on
   a. mathematics as problem-solving;
   b. mathematics as reasoning;
   c. mathematics as communication; and
   d. mathematical connections.

What examples of each curriculum focus are shown on the video?

3. How do teachers and other educators in the video think about the role and relevance of basic skills and procedures?

ORIENTING QUESTIONS FOR VIEWING THE VIDEO

Questions about the Teacher's Role

1. How do the teachers in the video describe what students in their classrooms are saying or doing differently as they learn mathematics?

2. What kinds of questions do teachers pose and how do they get all students involved in thinking about these questions?

3. How do teachers guide students' responses and how do they respond to students' unexpected ideas?

Questions about the Teacher's Role (cont.)

4. Teachers in the video use and describe different approaches to teaching mathematics (e.g., using manipulatives, alternative assessments, student writing, group work).

    a. Could teachers use these different approaches during mathematics instruction, yet remain tied to traditional mathematics teaching? For example, could a teacher use group work without teaching mathematics for understanding, or use manipulatives without any focus on helping them reason about mathematics?

Questions about the Teacher's Role (cont.)

   b. If so, what makes these different approaches consistent with teaching mathematics for understanding?

5. What difficulties do teachers and other educators describe in learning to teach mathematics for understanding?

   a. What do they describe as the benefits of trying to overcome these difficulties?

Questions about the Teacher's Role (cont.)

b. What obstacles might you confront if you want your students to engage in these activities? How might you manage these obstacles in order to help students focus on understanding mathematics?

MS. BALL'S 3RD GRADE CLASSROOM

Discussion questions:

1. According to the NCTM *Curriculum and Evaluation Standards*,
   "Students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. The situation should be complex enough to offer challenge, but not so complex as to be insoluble... Learning should be guided by the search to answer questions—first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)." (p. 10)

a. Do you think that the students in this class were trying to solve a "genuine problem"? What features of the problem make it genuine or not genuine?

b. How did Ms. Ball use this problem to engage students and elicit their mathematical reasoning and communication?

c. What could be a genuine problem for students at your grade level?

2. During the whole class discussion, the teacher rarely gave students any hint about whether she thought they were right or wrong, or even if they were on the right track. Why do you think she behaved this way? What effect did her behavior have on the students' reasoning about math?

3. One of the purposes for this lesson was to diagnose what students knew about fractions as they began a unit on fractions.

    a. Why might it be important for a teacher to learn what her students know about a topic they had not yet studied?

b. What did you find out about what students knew? Did anything surprise you?

c. Contrast this teacher's diagnosis of her students' knowledge of fractions with a more traditional pre-test approach.

4. Ms. Ball accepted a student's definition of the unit as "cutted bread." Would you do this? Is this good practice? What is your reasoning?

MS. BALL'S CLASSROOM (cont.)

5. According to the *Professional Teaching Standards*, "The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate."(p. 35)

MS. BALL'S CLASSROOM (cont.)

Orchestrating discourse depends "on teachers' understandings of mathematics and of their students—on judgments about the things that students can figure out on their own or collectively and those for which they will need input." (p. 36)

a. In what ways did Ms. Ball attend to these suggestions for orchestrating discourse?

b. Do you see places where a teacher might have made different decisions from those Ms. Ball made? What do you think her decisions were based upon? On what are your decisions based?

MS. BALL'S CLASSROOM (cont.)

6. The students in this class seemed comfortable reasoning about mathematics in a whole group setting, including at times disagreeing with their classmates. How might a teacher establish these norms of communication in her class?

MS. JONES'S
7TH GRADE CLASSROOM

Discussion questions:

1. The video segment shows students representing numbers with base ten blocks. According to the *Curriculum and Evaluation Standards*,

   "Students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. The situation should be complex enough to offer challenge, but not so complex as to be insoluble... Learning should be guided by the search to answer questions—first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)." (p. 10)

MS. JONES'S CLASSROOM (cont.)

a. Do you think that the students in this class were trying to solve a "genuine problem"? What features of the problem make it genuine or not genuine?

b. How could the problem be recast to make it more genuine or less genuine?

c. How does this method of learning decimals compare with your textbook's treatment of decimals?

2. The students in this segment worked in groups, as they do regularly in Ms. Jones's class (and as is encouraged by the *Standards*).

   a. What is the rationale for group work? What benefits do you think students derive from working in groups?

   b. This class was filmed in the middle of the year. What norms of classroom discourse and behavior do you think Ms. Jones had to cultivate throughout the school year for students to be able to work together as they did in this segment?

c. During the class discussion Ms. Jones consistently pushed students to elaborate when they responded to her questions. Why do you think she did this? Compare and contrast how she asked questions with the more traditional questioning that seeks a right or wrong answer from students.

d. What other questions or different prompts can you think of to help students develop decimal number sense?

3. The *Professional Teaching Standards* ask teachers to attend to standards in four areas: selecting worthwhile tasks; creating a learning environment; orchestrating discourse; and engaging in thoughtful analysis.

   How did this teacher appear to be attending to any of these standards during the lesson?

MS. JONES'S CLASSROOM (cont.)

4. Four standards in the *Curriculum and Evaluation Standards* are common to all grade levels: mathematics as problem-solving; mathematics as communication; mathematics as reasoning; and mathematical connections.

   How did these students appear to be attending to any of these standards during the lesson?

MR. SHERBECK'S CLASSROOM

Discussion questions:

1. The students in this segment worked in groups.

   a. What is the rationale for group work? What benefits do you think students derive from working in groups? What are the drawbacks?

   b. This class was taped in the middle of the year. What norms of classroom discourse and behavior do you think Mr. Sherbeck had to cultivate throughout the school year for students to be able to work together as they did in this segment?

c. If you wanted your students to talk to each other like the students in Mr. Sherbeck's class, how would you model this kind of conversation? How would you give feedback to students that would encourage constructive comments and discourage off-task comments?

2. Why might it be important for students to write about their mathematical thinking?

3. A teacher could ask students to write about mathematics without necessarily contributing to their problem solving, reasoning, or communication skills. How can teachers use students' writing in ways that promote meaningful mathematical learning?

4. The *Professional Teaching Standards* ask teachers to attend to standards in four areas: selecting worthwhile tasks; creating a learning environment; orchestrating discourse; and engaging in thoughtful analysis.

   How did this teacher appear to be attending to any of these standards during the lesson?

MR. SHERBECK'S CLASSROOM (cont.)

5. Four standards in the *Curriculum and Evaluation Standards* are common to all grade levels: mathematics as problem-solving; mathematics as communication; mathematics as reasoning; and mathematical connections.

   How did these students appear to be attending to any of these standards during the lesson?

MR. LEHMAN'S
ALGEBRA ASSESSMENT

Discussion questions:

1. What are the advantages and disadvantages of this type of assessment, where students individually demonstrate their knowledge and understanding by orally defending their solutions to problems?

   Contrast what students must know and be able to do for this type of assessment with what they must know and be able to do for more traditional pencil and paper tests.

MR. LEHMAN'S ASSESSMENT (cont.)

2. Although the students prepare their solutions while working in groups, they were required to explain their solutions individually.

   What might be some advantages, or disadvantages, of allowing groups to demonstrate their solutions collaboratively in this type of assessment activity?

3. If classroom instruction is not consistent with the type of sense-making, conceptual questions that panelists ask students, then students cannot be expected to do well on the assessment.

   If you knew your students would be assessed by a panel like this, how would you prepare them?

   Where would you find problems for them to work on?

   How would you organize classroom discussions to model this kind of reasoning about mathematics?

MR. LEHMAN'S ASSESSMENT (cont.)

4. What logistical or technical problems might be associated with arranging this type of assessment?

   What alternatives might be used by teachers who might have difficulty gathering a large number of panelists, but who would like to give students opportunities to demonstrate orally their mathematical sense-making and problem solving abilities?